Approaching asteroseismology of $\delta$ Scuti stars: problems and prospects

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Abstract

The main obstacle in exploiting the frequency data of $\delta$ Sct stars is the difficulty in mode identification. The $\delta$ Sct oscillation spectra, unlike those of the Sun or white dwarfs, do not exhibit very regular patterns. Thus, the mode identification must rely on sophisticated methods, which involve combined multi-passband photometry and radial velocity data, with an unavoidable theoretical input from stellar atmosphere models. Moreover, there are serious uncertainties in theory of $\delta$ Sct stars that have to be solved. Mode identification and determination of global and internal structure parameters for $\delta$ Sct stars has to be done simultaneously. I describe in some detail the methodology and present some recent results we obtained concerning degrees of excited modes, global stellar parameters, and constraints on models of subphotospheric convection, as well as effect of rotational mode coupling.

Introduction

$\delta$ Scuti stars are one of the most intensively studied group of pulsating variables. In the HR diagram, they are located at the intersection of the classical instability strip with the main sequence, and somewhat above it. It was recognized many years ago that the pulsations of these objects, as other classical variables, are driven by the $\kappa$-mechanism acting in the He ii ionization zone. Excited are low-order p- and g-modes with periods ranging from 0.02 d to 0.3 d.

Over the last 20 years, many multisite observations of these stars were carried out by networks like DSN and WET. These campaigns have resulted in a growing number of detected oscillation frequencies. On the basis of these data several attempts were made towards construction of asteroseismic models of certain multimodal pulsators. One of such objects was XX Pyx, for which Pamyatnykh et al. (1998) tried to construct a seismic model without an $l$ identification from photometry or spectroscopy. Another example, $\theta^2$ Tau, is a binary system consisting of an evolved (primary) and a main sequence A-type (secondary) star (Breger et al. 2002), both inside the instability strip. The most multimodal and most promising object for asteroseismology of $\delta$ Sct stars is FG Vir. This star was studied by Guzik & Bradley (1995), Viskum et al. (1998), Breger et al. (1999) and Templeton et al. (2001). Recent large photometric and spectroscopic campaigns, organized in the years 2002-2004 by Breger et al. (2005) and Zima et al. (2006), increased the number of known independent oscillation frequencies of FG Vir to 67. In spite of all these efforts we still do not have a good seismic model for any $\delta$ Sct star. So far, not much has been learnt from these rich oscillation spectra. There are still problems with the identification of excited modes as well as large uncertainties in modelling $\delta$ Sct pulsation to exploit the frequency data for constraining stellar models. The most important aspects are: turbulent convection-pulsation interaction, effects of rotation, mechanism of mode selection, diffusion.

In this paper, I outline the method which gives simultaneously mode identification and constraints on stellar parameters and convection. I discuss also effects of uncertainties arising from the atmospheric models and, briefly, effects of rotational mode coupling on mode identification.
Mode identification

In the case of main sequence pulsators, the most widely used tools for mode identification are pulsation amplitudes and phases derived from observed variations in photometric passbands and in radial velocity. If effects of rotation can be neglected, the amplitude ratio vs. phase difference diagrams can lead to the \( \ell \) degree determination, and they are independent of the azimuthal order \( m \) and the inclination angle. As was shown by Daszyńska-Daszkiewicz, Dziembowski & Pamyatnykh (2003) (Paper I), in the case of \( \delta \) Sct variables the photometric amplitudes and phases are very sensitive to the treatment of convection in the outer layers. This is because in calculating these observables one has to make use of the complex parameter \( f \), giving the ratio of the local flux variation to the radial displacement at the photosphere. The \( f \) parameter is obtained in the framework of linear nonadiabatic theory of stellar oscillation and, in the case of \( \delta \) Sct stars, exhibits strong dependence on convection, as was already emphasized by Balona & Evers (1999). To avoid this problem, in Paper I we invented a method of simultaneous determination of the \( \ell \) degree and \( f \) parameter from multi-colour photometry and radial velocity data. The procedure consists of solving the set of observational equations for complex photometric amplitudes in a number of passbands, \( \lambda \),

\[
D_\lambda^\ell (\bar{\varepsilon} f) + \varepsilon_\lambda^\ell \bar{\varepsilon} = A^\lambda, \tag{1}
\]

where

\[
\bar{\varepsilon} = \varepsilon Y_m^\ell (i, 0),
\]

\[
D_\lambda^\ell = \frac{1}{4} b_\lambda^\ell \left( \frac{\omega^2 R^3}{GM} + 2 \right) \frac{\partial \log(F_\lambda | b_\lambda^\ell |)}{\partial \log T_{\text{eff}}},
\]

\[
\varepsilon_\lambda^\ell = b_\lambda^\ell \left[ (2 + \ell)(1 - \ell) - \left( \frac{\omega^2 R^3}{GM} + 2 \right) \frac{\partial \log(F_\lambda | b_\lambda^\ell |)}{\partial \log \log g} \right].
\]

Derivatives of the monochromatic flux, \( F_\lambda (T_{\text{eff}}, \log g) \), are calculated from static atmosphere models (Kurucz: Kurucz 2004, NEMO2003: Nendwich et al. 2004, Phoenix: Hauschildt et al. 1997). In general, they depend also on the metallicity parameter \([m/H]\) and microturbulence velocity \( \xi_t \). If the spectroscopic data exist, the above set of equations can be supplemented with the expression for the radial velocity (the first moment of line profile, \( M_{\lambda 1} \)),

\[
i \omega R \left( u_\lambda^\ell + \frac{GM}{R^3 \omega} v_\lambda^\ell \right) \bar{\varepsilon} = M_{\lambda 1}^\ell. \tag{2}
\]

In the above expressions, \( \varepsilon \) is the intrinsic mode amplitude, \( i \) is the inclination angle and \( b_\lambda^\ell, u_\lambda^\ell, v_\lambda^\ell \) are disc-averaging factors weighted by limb-darkening \( h_\lambda (T_{\text{eff}}, \log g) \). For the limb-darkening law we use the Claret nonlinear formula. Each passband, \( \lambda \), yields the right-hand side of Eq. (1). The radial velocity data yield the right-hand side of Eq. (2). Then, the system is solved by the least square method assuming trial values of \( \ell \). The \( \ell \) identification is based on \( \chi^2 (\ell) \) minimization and the quantities to be determined are: \( \bar{\varepsilon} \) and \( (\bar{\varepsilon} f) \). In Paper I we applied our method to three \( \delta \) Sct stars: \( \beta \) Cas, 20 CVn and AB Cas, for their dominant frequencies. To this end we used amplitudes and phases in four Strömgren passbands. In all cases the identification of \( \ell \) was unique. As an example, in Fig. 1 we plot the \( \chi^2 (\ell) \) dependence for one frequency observed in \( \beta \) Cas. In the two panels, the effect of using atmospheric models from different sources is shown. In the left panel, \( \chi^2 (\ell) \) was obtained adopting the Kurucz models, whereas in the right one, adopting the Vienna models (NEMO2003). The method works also in the case of multiperiodic pulsators. In Daszyńska-Daszkiewicz et al. (2005) (Paper II) we applied the method to the most multiperiodic \( \delta \) Sct star FG Vir. Combining the Strömgren \( vy \) photometry and radial velocity data for twelve modes, we arrived at a unique identification of \( \ell \) in six cases, and we obtained the constraint \( \ell \leq 2 \) in the other six.
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Figure 1: Values of $\chi^2$ as a function of $\ell$ for the 9.897 d$^{-1}$ frequency excited in β Cas, for $M = 1.95 M_\odot$ and three effective temperatures. In the left panel, $\chi^2$ was calculated with the Kurucz models and in the right panel with the NEMO2003 models.

The most important property of our method is that the identification of the spherical harmonic degree, $\ell$, is independent of any input from nonadiabatic pulsation calculations. Moreover, the method uses simultaneously photometry and spectroscopy by combining these data into the system of observational equations. For more details we refer the readers to Papers I and II.

Constraints on convection

The method outlined above constitutes also a way of inferring $f$ from observations. The value of $f$, describing the bolometric flux perturbation, is determined in the pulsation driving zone, where the thermal time scale is comparable with the pulsation period. It means that this parameter is sensitive to properties of subphotospheric layers which are poorly probed by the oscillation modes. In general, the $f$ parameter depends on: mean stellar parameters, chemical composition, stellar convection and opacities. Thus, the strong sensitivity of the $f$ parameter on convection in the case of δ Sct pulsators may be considered as an advantage. Once we know the empirical $f$ values, we can compare them with their theoretical counterparts, and obtain valuable constraints on convection in subphotospheric layers.

In Paper I, we succeeded in extracting the $f$ parameter from photometric observations for all studied δ Sct stars: β Cas, 20 CVn and AB Cas. We adopted Kurucz models of stellar atmospheres. The pulsation calculations were made assuming a simplistic approach: the mixing-length theory and the convective flux freezing approximation. In the comparison of empirical $f$ values with the theoretical ones calculated with various values of the MLT parameter $\alpha$, we met a problem in reproducing both the real and imaginary part of $f$ with the same value of $\alpha$. The general result was that the observed values of $f_R$ were close to those calculated with $\alpha = 0$, whereas the $f_I$ preferred higher values of $\alpha$. The disagreement appeared to be mostly correlated with the uncertainties in the atmospheric models, which I discuss in the next section.

In Paper II we applied the method of simultaneous extraction of $\ell$ and $f$ from observations for the most multiperiodic δ Sct star FG Vir. We relied on NEMO2003 atmosphere models.
Combining $vy$ Strömgren photometry and radial velocity data, we extracted the $f$ parameter for twelve frequencies and compared them with the theoretical values calculated assuming two different treatments of convection. The first one was the standard mixing-length theory and the convective flux freezing approximation, as in Paper I. As the second one, we considered a non-local time-dependent generalization of MLT by Gough (1977). In the first case the agreement was found for models with $\alpha \approx 0.0$, which is evidence that convection in the outer layers of FG Vir is relatively inefficient. In the second case, which includes convection dynamics, the agreement was possible also with larger values of $\alpha$, but smaller ones ($\alpha \leq 0.5$) were still favoured as can be seen from Fig. 2 (taken from Paper II).

![Figure 2: The empirical $f$ values (dots with error bars) and the theoretical ones calculated for four values of the MLT parameter $\alpha$, adopting a non-local, time-dependent formulation of MLT. The real and imaginary parts of $f$ are shown in the left and the right panels, respectively.](image)

Modelling of $\delta$ Sct type pulsation with time-dependent convection treatment can be found also in several other papers, e.g. Grigahcène et al. (2005), Dupret et al. (2005a,b).

Uncertainties from atmospheric models

To calculate pulsation amplitudes and phases of photometric and radial velocity variations, one needs input from atmospheric models. As mentioned in the previous section, these are the monochromatic flux derivatives over effective temperature, $\alpha_T$, and gravity, $\alpha_g$, as well as the limb-darkening law, $h_\lambda$. In Fig. 3 we can see how non-smooth derivatives $\alpha_T$, calculated from Kurucz models (left panel) can produce artificial minima of $\chi^2$ derived from our method for a dominant mode of FG Vir. Derivatives obtained from NEMO2003 models (right panel) are smooth and only one $\chi^2$ minimum appears. In this case, we show also the effect of microturbulence velocity, $\xi$, on the location of the minimum of $\chi^2(T_{\text{eff}})$. The non-smooth flux derivatives affect also the inferred values of $f$. This is illustrated in Fig. 4 where empirical $f$ values for $\beta$ Cas obtained using Kurucz and Vienna models are compared with theoretical ones calculated for five values of the MLT parameter, $\alpha$. We can see that in the case of Vienna models both real and imaginary parts of $f$ are reproduced with the models assuming inefficient convection ($\alpha \approx 0.0$).
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Figure 3: $\chi^2$ as a function of effective temperature for a dominant mode of FG Vir derived using Kurucz (left) and Vienna (right) models. The dot with the error bar shows $\log T_{\text{eff}}$ derived from mean colours. The y-axes on the right-hand side contain the temperature flux derivatives, $\alpha_T$, in the Strömgren vy passbands. In the right panel the effect of the microturbulence velocity on the $\chi^2$ minima is also shown.

Figure 4: Comparison of the $f$ values inferred from Strömgren photometry for β Cas with the theoretical ones calculated with various MLT parameters $\alpha$. The empirical $f$ values were obtained adopting Kurucz models (left panel) and Vienna models (right panel). In the right panel the effect of the microturbulence velocity on the empirical $f$ values is also shown.

Rotational mode coupling

The most important effect of moderate rotation is mode coupling (Soufi, Goupil & Dziembowski 1998). It takes place if the frequency difference between modes $j$ and $k$ is of the order of the angular velocity of rotation, and if the spherical harmonic indices satisfy the relations: $\ell_j = \ell_k + 2$ and $m_j = m_k$. As eigenfunctions for individual modes, we have to consider superpositions of all modes satisfying the above conditions. Hence, the photometric amplitude of a coupled mode is given by (Daszyńska-Daszkiewicz et al. 2002)

$$A_\lambda(i) = \sum_k a_k A_{\lambda,k}(i).$$
where the contribution from modes the in the non-rotating star is determined by the coefficients \( a_k \) which are solutions from perturbation theory. Now, the location of the mode on the diagnostic diagrams depends on the azimuthal order, \( m \), the inclination angle and the rotational velocity. We considered a stellar model with the following parameters \( M = 1.8M_\odot \), \( \log T_{\text{eff}} = 3.866 \) and \( \log L/L_\odot = 1.12 \), and the rotational velocity of about 70 km/s, which are appropriate for FG Vir. As an example, we consider the rotational coupling between \( \ell = 0 \) and \( \ell = 2 \) axisymmetric modes with frequencies 19.342 and 19.597 \( \text{d}^{-1} \), respectively. In Fig. 5 we show the position for coupled modes on the diagram with the Strömgren \( y \) passband and the radial velocity. The left panel refers to the solution dominated by the \( \ell = 0 \) component, whereas the right one to the solution dominated by \( \ell = 2 \). For discussion of other effects of rotation within the perturbative approach see e.g. Pamyatnykh (2003).

![Figure 5: The positions of rotationally coupled modes (small open circles) in the \( A_\nu/A_y \) vs. \( \varphi_{Vr} - \varphi_y \) diagram. We considered coupling between a close \( \ell = 0 \) and 2 pair at a rotation velocity of about 70 km/s in a stellar model with \( M = 1.8M_\odot \) and \( \log T_{\text{eff}} = 3.866 \). Filled symbols indicate the positions of pure \( \ell = 0, 1, 2 \) modes.](image)

**Summary**

I outlined results obtained in Papers I and II, where we proposed and applied the new method of simultaneous determination of the spherical harmonic degree, \( \ell \), and the nonadiabatic parameter \( f \) from multi-colour photometry and radial velocity data. We demonstrated that inferring \( f \) values from such observations is possible, thus identification of \( \ell \) can be done without a priori knowledge of \( f \). Our method combines photometry and spectroscopy, and it gives the \( \ell \) identification at the highest confidence level achieved up to now. Moreover, by comparing empirical and theoretical \( f \) values, the method yields constraints on mean stellar parameters and on properties of subphotospheric layers. In the case of \( \delta \) Sct stars, this is the treatment of convective transport. Inferred values of \( f \) are consistent with models calculated assuming rather inefficient convection (\( \alpha \leq 0.5 \)). The \( f \) parameter constitutes a new asteroseismic tool which is complementary to oscillation frequencies.

It is obvious that detecting more and more oscillation frequencies is of great importance, especially in the era of asteroseismic satellite missions. However, it seems that asteroseismology of \( \delta \) Sct stars will be served better if we focus also on those frequencies for which very accurate and simultaneous ground-based data from photometry and spectroscopy can be obtained.
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