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Twenty-four in auloi. Aristotle, *Met.* 1093 b, the harmony of the spheres, and the formation of the Perfect System

Our knowledge of Ancient Greek Music owes much to passing comments made by ancient authors who, at this time, had anything else in mind but conveying technical information to people not acquainted with the basics of contemporaneous musical life. This often leads to the frustrating feeling that a treasure is hidden in a particular passage, while all interpretations offered so far are not nearly satisfactory, and the key seems still missing. This paper is going to deal with such a statement by Aristotle, and if the hypothesis offered here is accepted it will consequently influence our view of a crucial chapter in the development of Greek music, both in theory and practice: the formation of the Greater Perfect System.

In a sequence of examples of an erroneous way of attributing metaphysical sense to numerical coincidences Aristotle mentions also the observation

> … ὅτι ἴσον τὸ διάστημα ἔν τε τοῖς γράμμασιν ἀπὸ τοῦ Α πρὸς τὸ Ω, καὶ ἀπὸ τοῦ βόμβυκος ἐπὶ τὴν ὀξυτάτην νεάτην ἐν αὐλοῖς, ἧς ὁ ἀριθμὸς ἴσος τῇ οὐλομελείᾳ τοῦ οὐρανοῦ. (*Met.* 1093 b)

> … that the distance is the same in the letters between alpha and omega, and from the *bómbyx* to the highest *nḗtē* in auloi, of which the number is equal to the wholeness of the heavens.

Since we know very little about the construction of the classical aulos, we are happy to learn that it incorporated, either as an instrument or as a class of instruments, some feature that could be, in some more or less obvious way, associated with a given numerical value. To find out more about the nature of this association we must submit nearly every word of the quoted passage to a close observation, eliminating prima facie possible interpretations out of the context of the statement.

Not even the number in question is entirely clear. It is usually assumed that only twenty-four, the number of letters of the Ionian alphabet, employed at that time in Athens, can be meant. Taken literally, however, the d i s t a n c e between A and Ω is certainly twenty-three. But this number is indeed little appropriate for mysticism; and in any case the notion of distance between letters makes very little sense, and we should expect rather the number of letters to be used. The notion of distance seems transferred carelessly from the context of the aulos to the alphabet. That it was inappropriate for both items of comparison is extremely unlikely: there would have been no motive to dismiss the natural concept of number in favour of that of distance. So we may take as a first hint that we should accept only an interpretation that involves a true distance, or interval, measured by the number twenty-four.

So twenty-four of what did Aristotle have in mind? At first glance it is most obvious to think of some interval taken as a measure for the compass of the aulos: twenty-four semitones give a nice double octave; and the octave itself consists of twenty-four quartertones, which are well attested to have been employed as a measure by a certain school of music theorists.[1] Yet if the single octave is envisaged, the specific connection with the aulos is hard to understand. The octave was associated with stringed instruments as well,[2] and certainly established as an abstract unit already in the fifth century: especially those engaged in number mysticism would not have had resort to auloi when they could simply have said that the *harmonía* consists of twenty-four.

Nor does the double octave explanation work, since the semitone is no likely measurement in a musical culture whose most revered style was defined by its quartertone steps. And what is more, and detrimental to the entire interval measurement hypothesis, the observation cited by Aristotle must have originated in Pythagorean context. But this precludes the notion of measuring by either semitones or quartertones altogether, since it was one main doctrine of Pythagorean music theory that the tone cannot be divided into equal parts (nor can any concordant interval).[3] The notion of simply adding intervals like distances, though deeply rooted in practice, came to be associated with the name of Aristoxenus, while Pythagoreans

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[1] See Barker 1989, 55 f n. 3.

[2] The note names are originally lyre string names covering (a seventh or rather) an octave (so described already in our earliest source, Philolaus, fr. 6a).

[3] Barker 1989, 73 n. 19 (but erroneously mixing up semitones and quartertones).

had to use multiplication, and consequently ended up with nasty *leímmata* and 'Pythagorean' commas.[4]

Secondly, there might be some set of twenty-four playable notes. But how should these be defined? Certainly at that time there were fewer than twenty-four basic notes on each aulos, even if both pipes sounded different notes over a certain range, and if, as in the professional instruments of the era in question, additional holes were available by a mechanism of rotating metal collars.[5] Many notes, however, were achieved not merely by opening or closing finger holes, but by half-stopping them to different degrees, or by manipulation of the reed.[6] Yet these techniques yield a great variety of shadings, and nothing like a definite set of countable notes.[7] Moreover a

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[4] Where we find confusion between both approaches (attributed to the Pythagorean Philolaus; see below), only microtones are concerned, and the composition of these to larger units is certainly not envisaged.

[5] Even late aulos finds, which sometimes have this many holes, are obviously not standardized as regards hole number or arrangement; cf. below n. 7.

[6] Cross-fingering, as employed on modern wind instruments, is of little or no use on an instrument with such large finger holes in relation to the main bore diameter as the Greek aulos invariably had. Achilles Tatius, 8.6.6 (κλείσας ὁ αὐλητὴς τὰς ἄλλας ὀπὰς μίαν ἀνοίγει μόνην, δι᾽ ἧς τὸ πνεῦμα καταρρεῖ), a passage cited by West (1992), 95, n. 73, as possible evidence for cross fingering (or rather 'covered fingering', cf. Byrne 2000) need not be pressed: the point is that there is just one sounding hole, as in the panpipe there is just one sounding pipe; so uncovered lower holes can remain out of consideration (according to the limited acoustical knowledge of the author, which in turn was well in accord with the large finger holes of the aulos).

[7] If we do not count shadings but stick to a 'dynamic' definition of a note as a location within the scalar system, we must assume that there could be produced at least the three sensible notes of a pyknon per finger hole: ἕκαστον γὰρ τρύπημα τῶν αὐλῶν τρεῖς φθόγγους, ὥς φασι, τοὐλάχιστον ἀφίησιν, εἰ δὲ καὶ τὰ παρατρυπήματα ἀνοιχθείη, πλείους (Procl., *in Alc.* 197, p.191.13–15 Westerink); except for the lowest holes, we must add, be it vent holes or main bore opening, which were not fingered at all and thus could not be half-stopped. Proclus talks about the evolved professional aulos, but since the enharmonic pyknon seems to be implied, at least the first part of his statement will accord with the practice of Plato's time. Taken literally, an aulos with 3 notes per hole and n dissimilar holes will give $3n+1$ notes with similar lowest holes, and $3n+2$ with different ones, but not $24=3n+0$. In any case the later extended professional aulos, especially with additional (side?) holes, cannot be assumed to have been standardized enough to make any canonical count of notes; virtuosi had their instruments made for their specific needs (cf. below, p. 88 with n. 104).

number of notes is again no distance; so this explanation meets also the reservations we had to make above.

Another interpretation, put forward rather tentatively by A. Barker,[8] builds on the fact that Aristotle says ἐν αὐλοῖς rather than ἐν τοῖς αὐλοῖς, and so might refer to the whole class of instruments. Since these came in several sizes, and Aristoxenus remarks that their overall range extends to over three octaves, Barker points to the fact that "a three-octave range in any one generic system of tuning will be covered in 22 notes, so that twenty-four notes will extend to a little over three octaves." Still we would expect an expression in terms of numbers, not distances. But graver objections must be raised. Firstly, it is implausible that auloi of different sizes worked together to produce one continuous scale. Such an arrangement would be natural in a musical culture where the instruments play together in orchestral formations. This was not the case in Ancient Greece. Even on those seemingly rare occasions where more than one aulos played at one time, these were obviously of the same type. But usually the aulos was a solo instrument, or accompanied one singer or a chorus. So we must rather expect different sizes of auloi to produce similar scales set to different pitches. And indeed neither ancient music theory nor notation ever described anything coming near a three-octave scale.[9] Even when in post-Aristoxenian times the fifteen-scale system of notation would, in some places, have allowed for systems covering three octaves, these remained split up into two two-octave scales sharing the notes of one octave. Consequently we cannot assume that in much earlier times such a system was constructed for the sole purpose of incorporating the notes of about five different types of instruments that never actually played together, and moreover that this system, although it has left no traces in later writings,[10]

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8 See above, n. 3.

9 Cf. Barker (1989, 60 n. 18) himself: "no Greek theorist seriously considers a musical scale extending beyond two octaves."

10 Aristoxenus mentions the three octaves not in connection with scales, but in the course of finding a sensible definition of a greatest concordant interval (*Harm.* 1.20, p. 25.14–26.8 Da Rios). For musical usage, he says, this is the double octave plus fifth, for neither the voice nor any melodic instrument extends over three octaves – even if this interval might (τάχα γάρ … ἂν ποιήσειε: nobody uses it!) be found between large and tiny auloi, between children and men, or between auloi played normally and in whistling mode (overblown; which we will discuss in detail below). The implication is precisely that none of those relations is musical in the sense that it could occur within one melody or piece (δεῖ δὲ τὴν διάτασιν ὁρίζειν ἑνός τινος ὀργάνου τόνῳ καὶ πέρασιν),

was at its time popular enough to invoke numerological speculation. For as we shall se shortly, all the other examples Aristotle cites involve no obscure numerical relations but numbers which were known either to everybody or at least to anyone who was acquainted with the basics of 'Pythagorean' philosophy.

Thus we are left with only one possibility: the number twenty-four must be related, in some way or other, to an interval expressed in Pythagorean manner, as a ratio. Still it cannot simply designate that interval in the sense that one of the extremes is twenty-four times the other, because this would give the wholly impossible range of four octaves and a fifth,[11] which exceeds any attested aulos range, with or without overblowing, and also that of the Greek scalar system in its most evolved state, which comprised only three octaves and a minor third.[12]

Before we proceed with our argument we shall have a look at Aristotle's other examples, from which we can get an impression of the general way of thinking he criticizes. False emphasis has been put, he says, on the numerical coincidences that there are

- 7 vowels, 7 notes in the octave, 7 Pleiades, 7 against Thebes – and in the 7th year (some) animals lose their teeth,
- 3 consonances in letters (ΞΨΖ, because two sounds sound together in one letter), and 3 consonances in music (fourth, fifth, and octave, as the basic consonances),
- 8 and 9 the arithmetic and harmonic means, but 8+9=17 is the number of hexameter syllables (in holodactylic verses), which consists of two parts of 8 and 9 syllables respectively.

The final example is of special interest for us, both because it involves some explicit mathematics, also related to music, and because it is most intimately linked with the aulos statement, with which it forms part of the same sentence, depending on one initial λέγουσι δέ τινες, while separated from the previous ones by a remark of Aristotle.

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and thus not within one continuous scale. Just as ancient geometry regarded the straight line as extensible as far as needed but not extended beyond that point nor of infinite (though perhaps undefined) length, ancient music theory always kept its scales within the boundaries of musical practice.

11 This interpretation was indeed adopted by Reinach 1900, 447, without reference to the contradictory Aristoxenus passage (see above, n. 10).

12 The standard *sýstēma pentekaidekátropon* of the notation incorporates three octaves and a tone, but there were additional signs for another halftone.

Now the arithmetic and harmonic means are of course not fixed numbers, but variables to be found by fixed algorithms between any pair of given numbers. But there is good sense in equating them with the specific numbers 8 and 9, because these are the lowest whole numbers in which both means can be expressed, namely as arithmetic and harmonic means of the extremes 6 and 12. Wherever both means are calculated for any pair of numbers, the resulting series will be some multiple of that basic 6 : 8 : 9 : 12. That both these means are, on the other hand, treated as belonging together in such a way as they are, has an entirely musical cause: within the octave (1 : 2, or 6 : 12), those means build a framework of fifths (6 : 9 and 8 : 12) and fourths (6 : 8 and 9 : 12), with a whole tone in the centre (8 : 9). This structure happened to represent also the framework of what we can call the traditional standard lyre tuning (e–a-b–e; the gaps could be filled differently with the remaining three strings), so that in this case mathematics and music were found to blend exceptionally well. What we find implied in Aristotle's example may be nothing less than the core demonstration of Pythagorean thinking, as attested already in the celebrated Philolaus fragment.[13] But the next step is somewhat obscure: the same numbers that gained their meaning only as ratios are simply added, to yield a single number of syllables. Even if Aristotle's source had implied that the relation between both parts of the verse is 8 : 9 (which Aristotle does not stress at all), there must have been reference to the number 17, gained by that odd addition: numbers forming ratios have been treated as quantities.

This procedure, which displays more confidence in the ontological significance of certain numbers than mathematical sensibility, was perhaps quite typical for pre-Platonizing Pythagoreanism. At least Boethius attests that it was employed by Philolaus, for us the outstanding representative of this school of thought, because only he left a written account.[14] The conceptions and procedures outlined by Boethius are accepted as genuinely Philolaean by W. Burkert and M.L. West;[15] C.A. Huffman, in his recent commentary, however, argues for a later origin in the early Academy, namely Xenocrates.[16] Yet it seems hard to imagine how Platonic scholars should have returned to a much more primitive and obviously wrong account on microtones once Archytas had put forth his neat tunings.[17] In any

[13] Philol., fr. 6a.
[14] Boeth., *Inst. Mus.* 3.5, p. 276f Friedlein = Philol., fr. A26.
[15] Burkert 1962, 372–377; West 1992, 235f.
[16] Huffman 1993, 368–374.
[17] On these, cf. Barker 1989, 46–52; West 1992, 236–238; Franklin in this volume.

case, even if the Xenocrates hypothesis is accepted, the date of the account is early enough to serve as a background for Aristotle (who usually criticizes his contemporary colleagues from the Academy anonymously).

Whoever devised the system started from the traditional Pythagorean diatonic division of the fourth into two tones plus a *leîmma* or *díesis*, 8:9, 8:9, 243:256. This whole tetrachord can be expressed in whole numbers most easily as the sequence 192 : 216 : 243 : 256. Here the lower one of the two tones had to be expanded by 27 (216:243=8·27:9·27), the higher one by 24 (192:216=8·24:9·24). Consequently the differences between adjacent numbers in the series are also 27 (243–216) and 24 (216–192), and 27:24 is again a tone. In any case, such an importance was attached to the number 27 that it was accepted as 'the number of the tone',[18] and taken as a starting point of microtonal calculations, which involve a good deal of addition and subtraction, the details of which need not concern us here.[19] It will be noticed that the idea of attributing a single number to something that can be expressed only through a ratio reminds one of the way the numbers 8 and 9 are conceived as representing the arithmetic and harmonic means: whether Philolaus or an author from the early Academy is behind Boethius' account, he is doubtless thinking along the same lines as the unknown authority criticized by Aristotle.

Keeping this in mind, and having excluded all other possible explanations, we can now formulate a new hypothesis about the aulos statement, which plausibly stems from the same source: the 'distance' of 24 might be the difference between the extremes of a sequence of numbers that represent, in Pythagorean manner, some tonal structure inherent in auloi. Such a scheme would have its exact parallel in the way of determining the 'number of the tone' found in Boethius' account, and we would consequently attribute it to the same school, probably the non-Platonizing Pythagorean-

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[18] Cf. Plut., *Anim. procr.* 1018e. The conception of every single thing or idea having a definite number is just the literal application of Philolaus, fr. 4: καὶ πάντα γα μὰν τὰ γιγνωσκόμενα ἀριθμὸν ἔχοντι· οὐ γὰρ οἷόν τε οὐδὲν οὔτε νοηθῆμεν οὔτε γνωσθῆμεν ἄνευ τούτου, where ἀριθμὸν ἔχοντι may be translated by "have a number" as well as by the more sophisticated (but perhaps too Platonic?) "have number". For an even more simplistic conception, of gaining 'the number' by addition, cf. Anatolius in Iamb., *Theol. ar.*, 55,4f de Falco: ὁ ζʹ λέγεται τῆς πρώτης συμφωνίας ἀριθμὸς εἶναι τῆς διὰ δʹ, δʹ γʹ, ἀναλογίας τε γεωμετρικῆς αʹ βʹ δʹ. Similarly the references in Plut., *Anim. procr.* 1017e–1018a: the number of the *harmonía* is 35, which is 6+8+9+12, and at the same time 2^3+3^3. The source of these considerations might also be Philolaus.

[19] See West 1992, 235f; Huffman 1993, 367–374.

ism represented by Philolaus. In mathematical terms, we are searching for a musically meaningful sequence of numbers $a{:}b{:}c \ldots x{:}y{:}z$, for which $z-a=24$.

The first step to an identification of this sequence must be the identification of its extremes, labelled as '*the highest nḗtē*' and '*bómbyx*' by Aristotle. So which one is the highest *nḗtē* ? In the Greek scalar system, each *tónos* (key) had two or three *nêtai*, and at the end of the classical period there were already several keys in use.[20] So we might think of the *nḗtē* of the highest key available on a standardized type of modulating aulos. Fortunately – it might have been difficult to find the correct set of *tónoi* – any numerical rendition of the relationships of modulating scales will lead to numbers that are much higher than our 24.[21] No modulation of key is envisaged in the system we are looking for; which means that as an 'unmodulating system' it includes only one *mésē*.[22]

The most extended unmodulating system of ancient music theory is the so-called 'Unmodulating Perfect System', *sýstēma téleion ametábolon*; and we can expect that it includes former less complete models in all respects we are concerned with.[23] In spite of its name it incorporates the most common modulation to the neighbouring scale (situated one fourth above). But this type of modulation was not understood as a shift of *mésē*, because the system was perceived as combining the so-called 'Greater Perfect System' with the 'Lesser Perfect System', which share their tonal centres. So

[20] Cf. the harmonic structure of Philoxenus' 'Mysians' (which were accompanied by the aulos): West 1992, 364–366.

[21] If we take just the most archaic musical style with three *tónoi* (Lydian, Phrygian, and Dorian; cf. Ptol., *Harm.* 2.10, p. 62.18–20 Düring; ps.-Plut., *Mus.* 1134a; Ath. 635cd, p. 402.13–16 Kaibel; Bacchius 46, p. 303 Jan) separated by two tones, modulate only between two of them, and take those scales which are most easy to combine (Dorian and Phrygian), in their simplest conceivable form (i.e. not including the Dorian *hyperypátē* although it is part of Aristides' 'ancient Dorian', but just taking the central octave, while excluding, on the other hand, the Phrygian *nḗtē synēmménōn*, because it is not part of Aristides' 'ancient Phrygian'), and confining ourselves to the fixed notes of the systems, thus arriving at the certainly least complex modulating system, which not even exceeds the octave and can be described as d–e–g–a–b–d, we cannot assign to it lower numbers than 48 : 54 : 64 : 72 : 81 : 96, with a difference of 48 between the extremes. And in this simple system there is not even such a thing as a highest *nḗtē* because the *nêtai* of both *tónoi* are identical in pitch.

[22] Cf. Cleonid. 11, p. 201.14–202.5 Jan; Hagel 2000, 33–38.

[23] It does not include the *hyperypátē*; but that does not affect our argument.

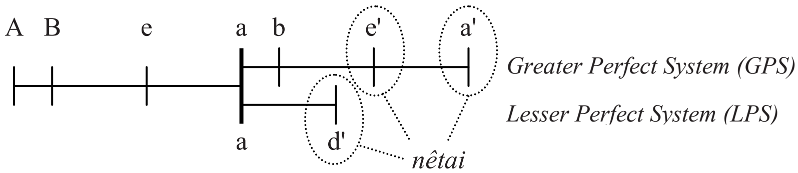


Figure 1: The Unmodulating Perfect System

the nomenclature is perfectly understandable, all the more since a similar 'modulation' is even inherent in the classical Phrygian scale as transmitted by Aristides Quintilianus. With modern note names indicating relative pitch,[24] the outlines of the *sýstēma téleion ametábolon* are given in Figure 1. For each tetrachord only the 'fixed' bounding notes are printed, not the two 'movable' notes whose varying positions indicated the genus of a scale.

This system contains no less than three *nêtai*: *nḗtē diezeugménōn* (e'), the highest note of the 'Dorian octave' so prominent in theory, *nḗtē hyperbolaíōn* (a'), the highest note of the system, and *nḗtē synēmménōn* (d'), the highest note of the modulating tetrachord. Taken literally, Aristotle's "highest *nḗtē*" as a superlative must refer to the highest of at least three, and so to *nḗtē hyperbolaíōn* (a'). But we should not be too sure. If his model system did not extend beyond *nḗtē diezeugménōn* (e'), the superlative might be induced by the fact that this is the highest note of all, and need not imply more than "the highest note, a *nḗtē*." So we are left with two possibilities, although the identification with *nḗtē hyperbolaíōn* (a') is in better accord with Aristotle's wording.

The second bounding note is called *bómbyx*. This term is associated with low, buzzing sound, and especially connected with the aulos.[25] There can be no doubt here that it designates the lowest note of this instrument, although we may not take it for granted that this is necessarily a playable note: early aulos pipes usually have, in addition to five finger holes, a vent hole below which ensures that the lowest note does not differ from the other ones in timbre, as is the case if it is produced from the whole tube.[26] Since βόμβυξ can refer to the pipe as a whole,[27] it presumably meant the

[24] Although the SOL-FA system would be more appropriate, for the convenience of most readers I have decided to use note names instead.

[25] Cf. Barker 1984, 187 n. 4; West 1992, 87 n. 30; 90 with n. 43.

[26] At the same time the pipe length is no longer defined by the requirements of the musical scale, and can be designed according to aesthetic considerations.

[27] Cf. Aeschyl., fr. 91: ὁ μὲν ἐν χερσὶν βόμβυκας ἔχων, τόρνου κάματον; Aristot., *Aud.* 800b; Theophr., *HP* 4.11.3. See Barker, as in n. 25.

note produced from this pipe, with all finger holes closed, rather than the voice of a vent hole.[28] It is not before the ps.-Euclidean *Division of the Canon* that we find *bómbyx* in close connection with a scalar step of the Perfect System:

> ἔστω τοῦ κανόνος μῆκος, ὃ καὶ τῆς χορδῆς, τὸ ΑΒ, καὶ διῃρήσθω εἰς τέσσαρα ἴσα κατὰ τὰ Γ, Δ, Ε. ἔσται ἄρα ὁ ΒΑ βαρύτατος ὢν φθόγγος βόμβυξ. οὗτος δὲ ὁ ΑΒ τοῦ ΓΒ ἐπίτριτός ἐστιν, ὥστε ὁ ΓΒ τῷ ΑΒ συμφωνήσει διὰ τεσσάρων ἐπὶ τὴν ὀξύτητα. καί ἐστιν ὁ ΑΒ προσλαμβανόμενος· ὁ ἄρα ΓΒ ἔσται ὑπάτων διάτονος.
>
> (*Sect. can.* 19)

> Let there be the length of the canon, which is also that of the string, AB, and let it be divided into four equal parts at the points C, D, and E. Thus BA, being the lowest note, will be the *bómbyx*. And the same AB is epitritic (4 : 3) to CB, so that CB will sound a concordant fourth above AB. And AB is *proslambanómenos*: so CB will be *diátonos hypátōn*.

Here the author first introduces the whole length of the string as AB, and infers that this length, just because it represents the lowest note, is the *bómbyx*. But why did it not suffice to say either that AB is the lowest note, or that it is *bómbyx*? That this equation is made at all implies that *bómbyx*, at that time, was commonly understood in a clearly defined sense: as the starting point of the division of a scale. Such a division is carried out in practice both by makers of fretted lutes and of wind instruments. Since the former are of virtually no importance in Ancient Greek culture, it is only natural that the theorists took over the respective terminology from aulos makers, who had to start from the *bómbyx*, the length of the entire pipe when marking out the proper places for finger holes.

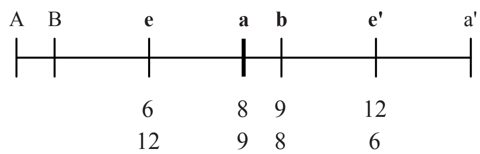
That AB is at the same time *proslambanómenos* (A in Figure 1), is no new information, as the simple ἐστιν instead of a defining ἔστω shows. Nor is it explicitly inferred from what preceded, which would have been indicated by the future ἔσται. Instead it is treated as having been said already, although we cannot be sure, where. Was it taken for granted that on the contemporary standard aulos the *bómbyx* sounded the *proslambanómenos*, as it indeed did on the best preserved instrument known as the Louvre aulos?[29] Or did it go without saying that the lowest note used in the division was at the same time the lowest note of the system to be established: that

[28] If the full tube length gave a usable note, too, vent holes could conceivable have been stopped e.g. with wax for certain types of music.

[29] Cf. Hagel 2004; see below.

there were no additional notes below *proslambanómenos* needed for the constructions to be carried out? In the latter case we might have expected at least an ἔσται, if not an ἔστω, but we cannot be sure. In any case we have to take what we get, and since there is no connection of *bómbyx* with any other scalar step but *proslambanómenos*, and since this connection is archaeologically corroborated, we have to investigate whether the same identification will make sense in the present context, too.

In a next step we have to establish plausible intermediate notes between both extremes. The structure we are looking for must lie on the way between the early Pythagorean division of the octave=*harmonía* and later full representations, which need higher numbers. The early division, as we have seen, gave values only for the framework of 'fixed' notes of what became the central octave of the Greater Perfect System:



Because the 'moving' notes have been left out, the simple sequence of fourth–tone–fourth has also the advantage of being symmetrical, and thus invariant to the direction of number assignment.

Of later more extended systems none became canonical. There were so many possibilities of combining or representing single genera and their microtonal shades, of displaying the system as a whole or just parts of it, of including or omitting the *synēmménōn* tetrachord. And all of them needed huge numbers, which were of no practical purpose and of no interest for numerological speculation. Still it is likely that there were intermediate steps: on one hand, the complete representation of the octave in one genus, presumably first in 'Pythagorean' diatonic; on the other, the traditional calculation of values only for the 'fixed' notes, but now for those of an extended system. The first possibility does not help us further: our system comprises certainly more than the octave; and not only the numbers resulting from the 'standard Pythagorean' tetrachord with its 256:243 *leîmma* are once more much too high, but also those for later, smoother divisions of the fourth (moreover none of these had ever acquired canonical status). But the construction of the 'fixed' notes of the Perfect System is well attested at least as a first step: In the *Division of the Canon*, chapter 19 is devoted to this procedure, while the moving notes are left for chapter 20.[30]

[30] On the similar procedure in Plato's *Timaeus* see below.

Nevertheless chapter 19 bears the title 'to construct the canon according to the so-called unmodulating system.'[31] Curiously enough, the 'moving' notes of chapter 20 are taken diatonically, while the propositions preceding the construction of the Perfect System presuppose the enharmonic genus; the transition is not mentioned at all. This incoherence strongly suggests that either chapter 19 and 20, or just chapter 20, are later additions. On the other hand it is hardly possible that a work known as, and obviously culminating in, the Division of the Canon, should not have contained it initially. Thus it is most plausible that only the last chapter was added, in a time when the need for a complete scale was felt.[32] The first author, who obviously had primarily the enharmonic in mind, could hardly set himself this task at all. While the enharmonic *likhanoí* and *paranêtai* could be constructed by concordant intervals (ch. 17), the *parypátai* and *trítai* could not; for these it was merely possible to 'prove' that they do not (as Aristoxenus claims) stand between two equal intervals (ch. 18). But such a purely negative demonstration provides no basis for a positive construction; so an enharmonic *Division* had to stop with the framework of 'fixed' notes. Such a framework could plausibly underlie also the scalar structure we are trying to reconstruct, especially since its date would be much nearer to, if not fall within, the time when enharmonic music flourished.

The last ambiguity we have to deal with is the direction of pitch in relation to numbers: ratios per se designate intervals without giving their direction, the interpretation of which relies on often unexpressed suppositions of physical or linguistic nature. Since Ancient Greek does by no means associate pitch with spatial arrangement, but with the notions of sharpness and weight,[33] in ancient sources ascending numbers are used both for ascending and descending pitch.[34] The division of a string, as on the canon, immediately leads to the association of higher numbers with lower pitch, just as wind instrument tube lengths and idiophone dimensions do. On the other hand, ancient physical theories usually connect a rise in pitch with a rise in sound force, speed or frequency; which view is based on experiences with string tension and vibration, wind instrument blowing

[31] Despite Barker's reservations (1989, 205 n. 65), the term 'unmodulating' is perfectly justified, because it stands for systems with one *mésē*, as opposed to those with more than one.

[32] For the single diatonic note in chapter 19 and its identification with *hyperypátē*, prominent in enharmonic scales, see Barker 1989, 205 n. 65.

[33] On the development of spatial connotations, cf. Rocconi 2002.

[34] Cf. van der Waerden 1943, 173–175.

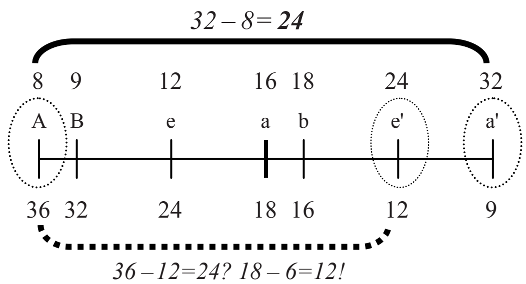


Figure 2: Assigning numbers to the Greater Perfect System

force, and the human voice, and linguistically expressed in terms of ἐπίτασις and ἄνεσις.[35] So we cannot know in advance which model Aristotle's source used, and have consequently to try both.

Now we can finally assign numbers to the Greater Perfect System, from *proslambanómenos* to both possible *nêtai*. The smallest numbers matching the interval relations of fourths and tones are those printed in Figure 2, both in rising and in falling direction.

To cover the double octave we need either the range from 8 to 32, in ascending order of pitch, or from 9 to 36, descending. On first glance, both sequences contain two steps between which we find a 'distance' of 24, and both occur between such notes as we have recognized as possible boundaries. *Proslambanómenos*, which we have equated with *bómbyx*, is always involved, while at the higher end of the scale it is either *nḗtē diezeugménōn* (e', 36–12) or *nḗtē hyperbolaíōn* (a', 32–8). But the former must be ruled out: if the scale in consideration extends not further than *nḗtē diezeugménōn* (e'), the series of numbers loses its only odd member, 9, and consequently is reduced to 18–16–12–9–8–6, with a distance between the extremes of only 12. At the same time it is clear that no subset of the notes smaller than the entire double octave of the Greater Perfect System will give the desired results.[36] So we find that the distance of 24 occurs exactly

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[35] The predominance of lyre types with equal string lengths combined with an almost complete absence of fretted lutes and the marginalization of pan flutes must have made string tension the primarily experienced pitch determinant, especially because everyone had to tune their lyres (but not to bore their auloi, for instance).

[36] The inclusion of *nḗtē synēmménōn* from the LPS into the scale does not work: only if we reduce the range to the central octave we get, in descending order, 24 : 27 : 32 : 36 : 48; but *hypátē mésōn* cannot have been *bómbyx* because it was used together with the degree three quartertones above already in the spondeion scale, at home in the most ancient auletic music we know of (cf. Winnington-Ingram 1928, West 1992, 206;

between those notes that we have established as the most plausible boundaries on altogether different grounds.

Let us consider briefly the results that our hitherto purely theoretical speculations have brought about. After the thorough exclusion of different explanations, we have set up as our working hypothesis that the number twenty-four represents the numerical difference between the extremes of an intervallic sequence. Subsequently we have proven that the only meaningful scale to match this requirement is the framework of 'fixed' notes of the Greater Perfect System. Both the inclusion of movable notes and the employment of modulating scales, even of the most reduced kind, lead to higher numbers, while, on the other hand, no smaller subset of the Greater Perfect System suffices. But are there any traces of such a numerical representation as we propose? And can we reasonably assume the Perfect System to have been established early enough, that is, not long after, if not before, 400 BC?

The first question finds its answer in a source from the middle of the second century of our era: Ptolemy's cosmic harmony, as laid down in his *Canobic Inscription*, and also in the lost final chapters of his *Harmonics*.[37] Underlying Ptolemy's approach is the traditional idea of connecting the celestial bodies, or spheres, with notes of a musical scale.[38] Ptolemy's conception seems however to have been original in certain ways. Though there are minor differences between the sources, which regard mainly the allocation of the lower spheres, Ptolemy's arrangement must have been that printed in Table 1.

Here the series of planets runs from the earth and its atmosphere, represented by the four elements, to the sphere of the fixed stars in a rather traditional manner, although the separation of the four elements at the cost

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Barker 1995, 50; Hagel 2004), and auloi did not have a finger hole so close to the end of the tube to produce that three-quartertone interval. If we extend the range to *hyperypátē* (obviously necessary in music of the late Classical era – cf. the Orestes fragment, DAGM 3 – and once also called *proslambanómenos*? Cf. below, p. 70 n. 56) we get distances of 15 (descending, with *nḗtē synēmménōn*), 20 (ascending), or 28 (descending, without *nḗtē synēmménōn*).

[37] On the sources and their variants, the problems of the arrangement, and the inclusion of the table in the *Harmonics*, see Jan 1894, 18–35 (who believes in a Ptolemaic origin only of the general thought), and now Redondo Reyes 2003 (with arguments for an astronomical derivation of the special features of the table) and Swerdlow 2004 (with special emphasis on its non-astronomical character).

[38] For an account of the different ancient cosmic scales, see Jan 1894; Richter 1999.

Spheres	Fixed Notes		Numbers
Fixed stars	† *mésē hyperbolaíōn*[41]	b'	36
Saturn ♄	*nḗtē hyperbolaíōn*	a'	32
Jupiter ♃	*nḗtē diezeugménōn*	e'	24
Mars ♂	*nḗtē synēmménōn*	d	21⅓
Sun ☉	*parámesos*	b	18
Venus, Mercury ♀☿	*mésē*	a	16
Moon ☾	*hypátē mésōn*	e	12
fire, air	*hypátē hypátōn*	B	9
water, earth	*proslambanómenos*	A	8

Table 1: Ptolemy's cosmic harmony

of attributing one single sphere to Venus and Mercury is unusual,[39] and underlines the analogical nature of the scale, which is obviously not intended to reflect astronomical facts accurately. This is the table as it has come down to us in the manuscript tradition of the *Canobic inscription*.[40]

In the *Excerpta Neapolitana* we find a more natural scheme, with Mercury and the Moon shifted down by one line, and the four elements united within the innermost sphere.[42] This same arrangement underlies another tradition, which has disposed of the note names and retained only the numbers and the names of the spheres, except that of the innermost one. This tradition is also united by the loss of the fractional part of the sphere

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[39] Usually either the planets are separated, or combined with the sun because of their equal average speed in relation to the fixed stars: so Plut., *Anim. procr.* 1029a9–b4 (on which see below); cf. *De defectu orac.* 430a; Plato, *Ti.* 38d; 36d. For further references, see Plut., *Anim. procr.* 1029b Cherniss 332f, note *c*.

[40] Ptol., *Inscr. Can.* p.154,1–10 Heiberg.

[41] This note name, for a degree absent from the usual systems treated in musical texts, is found in the *Canobic Inscription*, and evidently underlies μὲν ὑπερβόλαιον N, μὲν ὑπερβολαῖον P in the *Exc. Neap.* Since the scalar degree has nothing in common with *mésē*, it has been corrected into νήτη ὑπερβολαίων (of the key one tone apart) by Jan, into ὑπερυπερβολαία by Düring and Zanoncelli, into μετὰ ὑπερβολαῖον by Vincent. One might consider also ⟨παρα⟩μέση ὑπερβολαίων (the note above another disjunctive tone), or even ⟨προσλαμβανό⟩μεν⟨ος⟩ ὑπερβολαῖ⟨ος⟩ (cf. the passage from Plutarch cited in n. 44).

[42] *Exc. Neap.* §24, p.418.14–420.6 Jan.

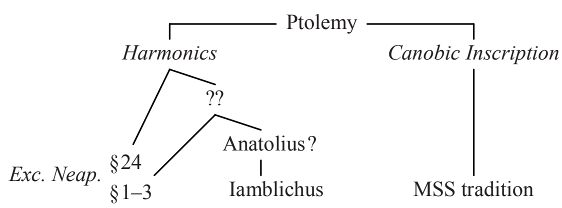
of Mars (*nḗtē synēmménōn*), where only the simple number 21 is retained, which has no scalar meaning. On the other hand this tradition was augmented by a rather tedious scholion-like explanation of the numerical relations inherent in the scale. This list is already based on the simple 21, for which it introduces the numerological conception of διπλασιεπίτριτος, $n\cdot(1+4/3)$, entirely alien to music theory. In this form Ptolemy's table appears in Iamblichus under the name of Anatolius, but perhaps as a later addition, and again in the *Excerpta Neapolitana*, but obviously without connection to the more original rendition there.[43]

The musical scale Ptolemy uses consists of the fixed notes of the Greater Perfect System, which is in perfect accord with the exposition of the lost chapter of his *Harmonics*, in which he must have explained its construction:

> κατὰ τίνας ἂν πρώτους ἀριθμοὺς παραβληθεῖεν οἱ τοῦ τελείου συστήματος ἑστῶτες φθόγγοι ταῖς πρώταις τῶν ἐν τῷ κόσμῳ σφαίραις. (Ptol., *Harm.* 3, p. 82.28–30 Düring)
>
> By the use of which smallest numbers it is possible to compare the fixed notes of the Perfect System with the primary cosmic spheres.

But we are surprised by an unusual topmost note, one tone above *nḗtē hyperbolaíōn*, which is never considered in musical sources.[44] It seems as if this note is introduced only to bring the total number of fixed notes into accord with the number of the spheres – or halfway into accord if we accept

43 Iamb., *Theol. ar.* 75,9–76,4 de Falco; *Exc. Neap.* § 1–3, p. 411 f Jan; the latter is little likely to stem from the former, since its heading as Πτολεμαίου μουσικά cannot have been inferred from the Iamblichus passage. Thus, the Greek evidence might be interpreted as suggesting the following affiliation:



This picture, however, can hardly be upheld against the Arabic evidence, which we will discuss below.

44 Cf. Redondo Reyes 2003, 184 n. 13: "sin duda este tipo de nuevas escalas debió de originarse en círculos astronómicos más que musicales …" The possibility of adding a *proslambanómenos* not at the lower, but (also) at the higher end of the scale is implied in Plut., *Anim. procr.* 1029c; see below.

the assignation of Venus and Mercury to a single sphere as original. And there is another oddity: while the inclusion of *nḗtē synēmménōn* in itself is not unexpected,[45] its numerical value does not fit the system, because it is written as a fraction, 21⅓. Obviously it has been inserted secondarily into a pre-existing sequence[46] without extending its numerical values; which procedure can be explained best if these values were already canonized by tradition. Even more so, because the resulting scale does not even fulfil Ptolemy's own claim to expound the πρῶτοι ἀριθμοί, the sequence of first, i.e. smallest whole numbers. Both additions, the highest note as well as *nḗtē synēmménōn*, are due to the somewhat violent connection of the traditional series with astrological speculations; and they are so obvious, that we can, by simply deleting them, reconstruct the underlying sequence with reasonable certainty. The resulting scale is identical to that one which we have postulated: a traditional numerical representation of the Greater Perfect System, running from 8 to 32.

The traditional state of these numbers might find further confirmation in the short statement that introduces the table in the Anatolius/Iamblichus tradition:[47]

> ἀρχὴ τῶν μουσικῶν λόγων ἐστὶν ὁ η´ ἀριθμός. καὶ εἰσὶν οἱ ὅροι τοῦ κοσμικοῦ συστήματος οὕτως…
>
> The musical ratios start from the number 8. And the boundaries of the cosmic system are the following…

While the second sentence is an adaptation of the original title of the table, ὅροι συστήματος κοσμικοῦ, the first phrase is not found in the *Canobic Inscription*. Nor is it taken from the lost chapter of the *Harmonics*, where Ptolemy uses the term μουσικός exclusively for persons, experts on music.[48] '*Lógoi mousikoí*' as ratios indicating musical intervals seem to belong to the realm of neo-Platonist and probably neo-Pythagorean terminology.[49] It is not altogether impossible that the phrase originated in the context where we find it, and simply indicates the starting point of the 'musical ratios' attributed to the celestial spheres. But in this case its placement be-

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45 Cf. Düring 1934, 281.

46 Cf. Redondo Reyes 2003, 188.

47 Iamb., *Theol. Ar.* 75,8–10 de Falco; *Exc. Neap.* §1, p. 411,2f Jan.

48 According to Düring's index, once to identify Didymus, thrice in superlative plural: (even) 'the most outstanding experts on music.'

49 Cf. Iamb., *in Nic.* 118,21; Procl., *in R.* 2,42,14; 2,80,18. On Plut., *Anim. procr.* 1029a, see below.

fore instead of after the title is very odd, especially because the number eight is naturally perceived as belonging to the 'boundaries' which are announced there. Therefore the statement about the number eight as the basis for 'the musical ratios' plausibly comes from another source: it might have been 'Pythagorean' commonplace, associated with our numerical sequence for the fixed notes of the Perfect System. Admittedly this is speculative; but in any case we can be certain that our scalar series existed, and was obviously regarded as canonical, by the time of Ptolemy.

It seems curious that this system of all models of assigning notes (and numbers) to celestial spheres did not make it into Latin writings, and so into the European Middle Ages. Perhaps the domination of the fixed notes, so crucial for the genera and modulations of Greek music, was not very attractive for Latin writers. We know nothing about native tonal systems of ancient Europe apart from the mainstream of Greek musical culture; but the increasing popularity of the diatonic genus in the fragments of the Roman period, as well as its final triumph in European music, especially of the Latin Church, might indicate that the melodies Latin writers were most familiar with were basically diatonic heptatonic. In such a musical environment the notion of fixed boundary notes of tetrachords was of not much use; and the Greek system, though transmitted by theorists in the absence of a generic one, remained foreign.[50]

But there may be a trace of our scale, and possibly in its pre-Ptolemaic version, in Arabic medieval writing. Noel M. Swerdlow kindly draws my attention to the *Epistle on Music*, part of the encyclopaedic work going under the name of the Ikhwān al-Ṣāfā'.[51] There the cosmic scale is introduced by the following statement:

> On the subject of the virtue of the number eight, the mathematician philosophers[52] have advanced the theory that a harmonious proportion exists between the diameters of the celestial spheres and those of earth and air. (Shiloah 1978, 45)

[50] If, as seems likely, Boethius had planned to adapt the third book of Ptolemy's *Harmonics* (cf. Mathiesen 1999, 635), his account of celestial music would almost inevitably have included the fixed notes scale.

[51] 'Brethren of Purity'. The epistles are held to have originated in 10th century Baghdad. I have used the English translation provided by Shiloah (1978). Cf. Swerdlow 2004, 178.

[52] In a different context, the Epistle mentions, in this sequence, Pythagoras – Nicomachus – Ptolemy – Euclid – "and other philosophers" (Shiloah 1978, 38).

The separation of earth and atmosphere into two spheres is that of Ptolemy's system, but the argument in general is more related to the Anatolius/Iamblichus tradition, where the cosmic system is not constructed for its own sake, nor to establish the connection between human and celestial music, but in purely numerological context, as an example of the importance of the number eight. Again we must emphasize that this thought cannot stem from the lost chapters of Ptolemy's *Harmonics*. Not only was this author too scientific a mind to attribute such a sort of 'meaning' to simple numbers; his work on music simply did not provide the context for such a numerological argument as implied by the *Epistle*. Consequently, the Ikhwān al-Ṣāfāʾ seem to rely on the same source as Anatolius. Their numbers for the individual spheres are quite unusual: earth 8, air 9, Moon 12, Mercury 13, Venus 16, Sun 18, Mars 21½, Jupiter 24, Saturn 24⁴⁄₇, fixed stars 32.[53] The explicit statement that the maleficent planets, Mars and Saturn, stand in no harmonic relationship to the rest[54] is undoubtedly related to Ptolemy's principles, though the system in which it is presented is not Ptolemy's, but evidently more primitive. We notice that the Ikhwān al-Ṣāfāʾ sequence extends only to 32, just as the original series postulated by us. Is it possible that it is nevertheless based on Ptolemy's arrangement, clipped and contorted? No, for without the explicit background of the canonized Perfect System, it makes no sense at all to abandon the upper limit of 36, so beautifully invoking the partition of the celestial sphere of 360° into 12 zodiacal signs, divisible into three parts each, in favour of the number 32, associated only with squares and cubes. And what is at least equally significant, those and only those numbers are common to Ptolemy's tables and the Arabic system that belong to our sequence of values for the fixed notes. On the other hand, the authors diverge in all the additional numbers and in the assignment of three celestial spheres. So it is very plausible that this Arabic system draws not on the *Harmonics* but on the same source as Ptolemy: that source which explicitly attached importance to the number eight as the starting point of musical ratios. This text might have considered how to extend the fixed notes series to incorporate all the planets and proposed a solution, which was however not accepted, and heavily modified either (and most probably) by Ptolemy or by the Ikhwān al-Ṣāfāʾ writer or by both of them.

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53 Shiloah 1978, 45.
54 Shiloah 1978, 46.

So there is plenty of evidence that our proposed numerical rendition of the Perfect system was indeed known in antiquity, and well-established by earlier than the middle of the second century of our era. But can we assume that it was already known 500 years earlier? Certainly it must be said that of what there was of systematic treatments of musico-numerical relationships we know little, and can read less. Only scraps of Philolaus' work have survived, while the Division of the Canon does not use numerical values where it can help it; nor can we expect ratios from Aristoxenus. So most depends on the date of the establishment of the *proslambanómenos*: with this note, which we have equated with the *bómbyx* of the Aristotle passage as a working hypothesis, the Perfect System was completed. But it may be doubted whether this step was taken so early. On one hand the note name is no longer connected with lyre string names, as are all the other ones.[55] Yet we have to take into account that the names of all notes of the Perfect System other than those of the central octave (except *hyperypátē* which did not make it into the system) are derived not from the practice of lyre tuning, but from a sophisticated tetrachordal abstraction of a standardized tuning. The *likhanós hypátōn*, for instance, derived its name not from a new string which was plucked with the forefinger, as was the original *likhanós*, now tagged '*mésōn*', but because both are the highest but one notes of their respective tetrachords. Only the *proslambanómenos* had no functional counterpart. Though it corresponds to *mésē* insofar as it lies below a disjunction, one could hardly term the outermost tone some kind of 'middle note'; and though it lies to the tetrachord *hypátōn* as old *hyperypátē* to the tetrachord *mésōn* there was no convenient way to transfer this name: just as the *hyperypátē* could not be included in the system because it competes with the tetrachord *hypátōn*, in the range of which it lies, while its name is oriented on the tetrachord *mésōn* above, it was impossible to coin a term like *hyperypátē hypátōn*, 'beyond-*hypátē* of the *hypátōn* tetrachord', for a note not belonging to this tetrachord.[56] On the other hand,

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[55] Cf. West 1992, 222.

[56] If Plut., *Anim. procr.* 1028f–1029a, is not simply a mistake (but should we not trust Plutarch to know the Perfect System?), but the terminology taken over from the source, the term *proslambanómenos* was once applicable to *hyperypátē*: which must have been before the names for the degrees of the Perfect System had become canonical. The passage seems to describe the well-known 'Phrygian' cosmic scale, but apparently in its diatonic variant instead of the usual chromatic (cf. Reinach 1900, 442f; Burkert 1961, 31–43; Hagel 2000, 46f; Burkert, p. 39 n. 2, as well as Richter 1999, 300, try to explain the well-attested scale away, but overlook its connections to Classical and

once the Perfect System was established by appending two extra tetrachords on each end of the old *harmonía*, resulting in a range of an octave plus seventh, its extension to the neat two-octave system was plausibly realized rather soon, if not immediately. Accordingly, the term *proslambanómenos* cannot serve as an argument for the relative lateness of the associated scalar degree.

Still there is a statement by Plutarch that might well be interpreted as indicating a late origin of the *proslambanómenos*. The entire passage is of such importance for our study that we have to print it in full:

> οἷς δ᾽ οὖν οὐ δοκεῖ ταῦτα τῆς τοῦ Πλάτωνος ἀπηρτῆσθαι διανοίας, ἐκεῖνα κομιδῇ φανεῖται τῶν μουσικῶν λόγων ἔχεσθαι, τὸ ε΄ τετραχόρδων ὄντων τῶν ὑπάτων καὶ μέσων καὶ συνημμένων καὶ διεζευγμένων καὶ ὑπερβολαίων ἐν πέντε διαστήμασι τετάχθαι τοὺς πλάνητας· ὧν τὸ μέν ἐστι τὸ ἀπὸ σελήνης ἐφ᾽ ἥλιον καὶ τοὺς ὁμοδρόμους ἡλίῳ, Στίλβωνα καὶ Φωσφόρον· ἕτερον τὸ ἀπὸ τούτων ἐπὶ τὸν Ἄρεος Πυρόεντα· τρίτον δὲ τὸ μεταξὺ τούτου καὶ Φαέθοντος· εἶθ᾽ ἑξῆς τὸ ἐπὶ Φαίνωνα καὶ πέμπτον ἤδη τὸ ἀπὸ τούτου πρὸς τὴν ἀπλανῆ σφαῖραν· ὥστε τοὺς ὁρίζοντας φθόγγους τὰ τετράχορδα τὸν τῶν πλανωμένων λόγον ἔχειν ἀστέρων. ἔτι τοίνυν τοὺς παλαιοὺς ἴσμεν ὑπάτας μὲν δύο τρεῖς δὲ νήτας μίαν δὲ μέσην καὶ μίαν παραμέσην τιθεμένους, ὥστε τοῖς πλάνησιν ἰσαρίθμους εἶναι τοὺς ἑστῶτας. οἱ δὲ νεώτεροι τὸν προσλαμβανόμενον, τόνῳ διαφέροντα τῆς ὑπάτης, ἐπὶ τὸ βαρὺ τάξαντες τὸ μὲν ὅλον σύστημα δὶς διὰ πασῶν ἐποίησαν, τῶν δὲ συμφωνιῶν τὴν κατὰ φύσιν οὐκ ἐτήρησαν τάξιν· τὸ γὰρ διὰ πέντε πρότερον γίγνεται τοῦ διὰ τεσσάρων, ἐπὶ τὸ βαρὺ τῇ ὑπάτῃ τοῦ τόνου προσληφθέντος. ὁ δὲ Πλάτων δῆλός ἐστιν ἐπὶ τὸ ὀξὺ προσλαμβάνων· λέγει γὰρ ἐν τῇ Πολιτείᾳ τῶν ὀκτὼ σφαιρῶν ἑκάστην περιφέρειν ἐπ᾽ αὐτῇ Σειρῆνα βεβηκυῖαν…
>
> (Plut., *Procr. anim.* 1029a–c)

> Now, for those who do not think that this (previously related account) is dependent on Plato's thoughts, the following will appear to stick entirely

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Hellenistic musical practice). Does the use of the Phrygian scale, associated with aulos music, point to a school of theorists more concerned with the aulos than the mainstream of our extant sources? Perhaps the scale was chosen just for its convenient layout: of all archaic scales transmitted by Aristides Quintilianus only his Phrygian unites nine degrees (needed for the seven planets, the earth, and the fixed stars) within a single octave, and this so that the tonal centre, *mésē*, is neatly occupied by the Sun. At least the inventors of this equation were not bothered by preconceptions demanding a cosmic priority for the 'most noble' Dorian.

> to musical reasoning (musical ratios?): that there are five tetrachords, *hypátōn*, *mésōn*, *synēmménōn*, *diezeugménōn*, and *hyperbolaíōn*, and at the same time the planets are arranged in five intervals, the first of which is from the Moon to the Sun and those running at equal speed with the Sun, Mercury and Venus. The next one from these to Fiery Mars. The third one between him and Jupiter. Next in order, that to Saturn, and the fifth one is already that one from him to the fixed sphere. Consequently the boundary notes of the tetrachords have the ratios of the planets. Now we know that already the ancients had defined two *hypátai*, three *nêtai*, one *mésē*, and one *paramésē*, so that there was the same number of fixed notes as of planets. But those who came later put the *proslambanómenos*, which is at the distance of one tone from *hypátē*, at the bottom, thus making the whole system a double octave, but failing to maintain the natural order. For the fifth is produced in advance of the fourth if the tone is added from *hypátē* downwards. But Plato obviously added it upwards (from the top): for he says in the *Republic* that the eight spheres revolve with a Siren standing on each of them…

The argument proceeds by reminding us that these Sirens sing one note each. There is besides a tradition of nine Muses, eight in the heavens, but one on earth. It is left to the reader to conclude from this rather enigmatic reference that Plato added a '*proslambanómenos*' at the top of the scale. The implication is that if there are eight spheres, but only seven original fixed notes, Plato has obviously used some sort of *proslambanómenos*. But if there is a ninth Muse, attached to the disordered realm of the earth, it is to her that in the full scale of nine Muses = Sirens the usual *proslambanómenos* must be assigned. Accordingly, the additional note of the celestial harmony, Plato's 'true' *proslambanómenos*, resided at the top end of the scale. We will come, in a moment, to the problems that Plutarch's references to the non-Platonic systems pose. For what Plutarch takes to be Plato's celestial harmony, we have all the information we need. There are seven degrees reserved for the planets, so each planet will get a sphere of its own; the degrees are the fixed notes, and we have to add a celestial and a terrestrial *proslambanómenos*, so that the earth falls out of the harmony of the double octave. The resulting scheme is printed in Table 2, where the order of the planets is, of course, that of the *Republic*.

Since Plutarch regards the mathematical implication of a system as the criterion of its admittance as a valid interpretation of Plato, and the identity of musical and planetary ratios is the only mathematical association mentioned, we would be justified in adding, as in Ptolemy's table, numbers establishing the ratios. But although the ratios are identical to those found in the *Canobic inscription* we cannot be sure whether the absolute numbers Plutarch had in mind, if any, were the smallest whole numbers, or included the irregular 21⅓ for Mars.

So far Plutarch has taught us three important things. Firstly, that he was already aware of the fixed notes version of the cosmic scale, and moreover already in exactly the same extension of two octaves and a tone as we find it in Ptolemy. Secondly, that he found it plausible that such a scale was already considered by Plato – though this admittedly does not mean much; as a Platonist, Plutarch would readily attribute everything to the divine philosopher. But thirdly, that, although the introduction of the *proslambanómenos* is attributed to some νεώτεροι, Plutarch obviously regards these as prior to Plato, for whom he not only claims the usage of some idiosyncratic high *proslambanómenos*, but also the philosophically 'impure' traditional one in the full scale that includes the earth. We do not know how much of Plutarch's account of the completion of the Perfect System is only speculation (perhaps everything); so we cannot infer a pre-Platonic date from his words. But he is certainly not a witness against an early Perfect System.

Fixed Notes		**Spheres**
'proslambanómenos'	b'	Fixed stars
nḗtē hyperbolaíōn	a'	Saturn
nḗtē diezeugménōn	e'	Jupiter
nḗtē synēmménōn	d'	Mars
paramésē	b	Mercury
mésē	a	Venus
hypátē mésōn	e	Sun
hypátē hypátōn	B	Moon
proslambanómenos	A	Earth

Table 2:
Plutarch's Platonic cosmic harmony

Much more problematic is Plutarch's account of the systems which he seems to associate with the παλαιοί.[57] What he says cannot possibly apply to one single system; but we can excuse the author on the assumption that he only wished to recall very briefly some data he needed for the reconstruction of the Platonic system. The notion of five tetrachords resembling five intervals between spheres can never have been anything more than number play of the crudest sort. It can be transposed into an actual sequence only on the assumption that these tetrachords formed a continuous series, which they do not. The bifurcation of the system at *mésē* must destroy any hope of reconstructing such a sequence: the five tetrachords actually establish six intervals (with seven border notes), too much to fit Plu-

[57] In the context of music theory and practice, largely received through the writings of Aristoxenus, this term might well refer to the mid fifth century.

tarch's list of five intervals into.[58] With the sentence about the ratios between the fixed notes being those of the planets (and not the other way round: human music is an image of the cosmic scale) we are already within the paradigm as we know it from Ptolemy, concerned with the framework, not the intervals.

Plutarch has led us at least one step further. His awareness of a scale of fixed notes, and its ratios resembling those of the universe, is the final proof that it is a good deal older than Ptolemy. If Plutarch is able to attribute such a system not only to Plato but even to earlier theorists, it cannot have been a novelty by his time: either its true origins were lost in the mists of time, or Plutarch really knew about a fifth or early fourth century source for the numerical series, just as our reconstruction would demand – except for the inclusion of *nḗtē synēmménōn*, which is still absent from our sequence, and, considering its fractal appearance even in Ptolemy's tables, has carried the stigma of a newcomer for centuries. So we have now arrived at a plausible date in rather early Hellenistic times; and we can proceed to pose a final question: might it be possible that Plutarch is actually right in attributing the knowledge of the scale, along with its numerical rendition, to Plato's *Timaeus*?

There the creation of the cosmic soul involves the partition of its 'matter' according to numerical values.[59] Though it is denied that the result is immediately audible,[60] no one has ever doubted that Plato has the musical model in mind, especially as in a final step the sequence is filled by 'Pythagorean' tetrachordal tuning as first described by Philolaus. The previous steps are taken as follows:

1) Starting from the number one, two sequences are established, one by duplication, one by triplication. Both are extended to the cube of their first number (different from unity):

$$1 \begin{array}{l} \nearrow 2-4-8 \\ \searrow 3-9-27 \end{array}$$

[58] I do not really understand how Richter 1999, 302, arrives at his reconstruction, but at least in separating the Sun from its ὁμόδρομοι it is at odds with Plutarch's text, where they are certainly conceived as sharing their place with the sun: even if the phrase ἐφ᾽ ἥλιον καὶ τοὺς ὁμοδρόμους ἡλίῳ, Στίλβωνα καὶ Φωσφόρον could conceivably be understood as carelessly passing over one interval, the following ἕτερον τὸ ἀπὸ τούτων seems indisputable. Cf. Plut., *Anim. procr.* 1029b, Cherniss 233, note *g*.

[59] Plato, *Ti.* 35b–36b.

[60] Plato, *Ti.* 37b.

2) The harmonic and arithmetic means are inserted between neighbouring numbers (to avoid fractions, everything has to be multiplied by six):[61]

6 < *8 – 9* – **12** – *16 – 18* – **24** – *32 – 36* – **48**
6 < *9 – 12* – **18** – *27 – 36* – **54** – *81 – 108* – **162**

3) The resulting values are then conceived as forming one sequence:
6 – 8 – 9 – 12 – 16 – 18 – 24 – 27 – 32 – 36 – 48 – 54 – 81 – 108 – 162

All values that are needed for the standing notes of the Greater Perfect System are already present in the series of even numbers of step (2). But since the orientation of pitch in relation to number is unclear, the full sequence can be interpreted in terms of music in more than one way:[62]

6	8	9	12	16	18	24	27	32	36	48	54	81	108	162
(E)	**A**	**B**	**e**	**a**	**b**	**e'**		**a'**	(b'	e''	f♯'	c♯''	f♯''	c♯''')
(e''	b')	**a'**	**e'**	**b**	**a**	**e**	**d**	**B**	**A**	(E	D	G'	D'	G'')

Thanks to its nearly symmetric arrangement round the central tone of disjunction (a–b, 16 : 18), the Greater Perfect System is present in Plato's construction regardless of the direction in which one prefers to read it. Which direction had Plato in mind? I am very much inclined to think, both. For when he proceeds to the insertion of diatonic 'movable' notes into this framework, he fails to mention the direction in which to carry it out, so that no final system can be reconstructed unambiguously. Plato develops a cosmic harmony which extends well beyond the limitations of human music, and which does not simply correspond to earth-bound sound. Yet the whole system of human music, insofar it is governed by invariable principles, symbolized by fixed as opposed to movable notes, is present as a subset of the universal order. There is little doubt that Plato was well acquainted not only with music theory of Pythagorean hue, and with Damon's thoughts, but also with the most recent developments of pre-Aristoxenian harmonists. Certainly he does not deem the technicalities of any of these schools worthy of philosophical discussion; but he proves himself able to play with them ingeniously. Those of his readers familiar with the numerical se-

[61] This is not told by Plato, who is not concerned with absolute numbers (except when he finally gives the ratio of the *leîmma*, of course in whole numbers), but naturally done by commentators; cf. e.g. Plut., *Anim. procr.* 1019 bc: ἐδέησε μείζονας ὅρους λαβεῖν ἐν τοῖς αὐτοῖς λόγοις; Procl., *in Tim.*, 2, p. 170,32 αὐξήσαντες τοὺς ἀριθμούς.

[62] In order to render identical intervals by identical distances, the numbers in the diagram are arranged logarithmically.

quence we have postulated would easily recognize it in his construction, and admire the elegant way he has devised it.

A closer inspection of Plato's numbers might reveal another level of subtlety. While *nḗtē synēmménōn* (d') is entirely absent, we meet, in the 'falling' rendition, its counterpart at the octave, *hyperypátē* (d). This note, as far as we know, played a role in classical music both in the Phrygian and in the Dorian mode. *Nḗtē synēmménōn*, on the other hand, was part only of the Phrygian modal scale, or some variants of it,[63] and it was used for the most basic type of modulation. Note that *hyperypátē*, irregular in the context of the later canonical system, but conceivably essential for classical Dorian, the mode most highly esteemed by Plato, is included, while the modulating *nḗtē* is banned from the construction, just as modulating music is from the ideal state.[64] This may be by chance, however, simply because when Plato devised a system incorporating all the numbers he needed in a mathematically 'unsuspicious' way, he had to take those, too, which happened to come along. And perhaps he was just content if the single number interrupting the standard fixed notes sequence had any musical meaning at all.[65]

In any case it is clear that Plato composed the division of the cosmic soul against the background of music theory, conceived in a Pythagorean way; that the system he proposed includes all the numbers needed for the fixed notes of the two-octave system; and that the final step of the division presented these two octaves in a full heptatonic shape, in diatonic 'Pythagorean' intonation. It seems a safe conclusion, therefore, that Plato already knew the Perfect System, and most probably also the numerical series for its fixed notes. Otherwise one would have to assume that the Perfect System is present in Plato's structure by pure chance; hardly a plausible alternative. Consequently we have established the required pre-Aristotelian date for our numerical sequence.[66] Might the ultimate source be sought al-

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63 It is part of the Aristides scale, but not treated as indispensable for Phrygian by ps.-Plutarch, *Mus.* 1137cd (most probably from Aristoxenos).

64 Plato, *Rep.* 397b–d; 399cd.

65 Curiously enough, the same number 27 which is alien to our sequence is missing in the manuscripts of Proclus' commentary (*in Tim.* 194b, vol. 2, p. 176.26 Diehl).

66 It is worth noticing that von Jan, in the first modern account of the 'fixed' notes cosmic scale, already feels, in spite of his belief in a Ptolemaic origin, that the thought ought to have a history dating back to Plato: "Es ist merkwürdig, daß dieser im Grunde doch schon von Plato angeregte Gedanke erst in der Verfallzeit des Griechenthums weiter verfolgt und zur Begründung eines Sphärensystems verwendet wurde" (Jan 1894, 28).

ready in Philolaus' work? Both the *Timaeus* and the passage from Aristotle point to an origin in pre-Platonic Pythagorean thought, and the lack of scientific method implied by Aristotle, the subtraction of numbers forming a ratio, recalls the way of treating numbers that Boethius ascribes to Philolaus. So one might suspect that the development of the fixed notes framework may conceivably have formed the intermediate step between the basic structure of the octave and the description of the 'Pythagorean' tetrachord tuning, just as the empty tetrachordal structure is first established, then divided in the *Timaeus* and in the *Division of the Canon*. But there is a serious objection: the fixed note series proceeds from lower to higher pitch, while in Philolaus' tetrachord the highest number corresponds to the lowest note. Unless we assume that Philolaus switched between both models according to which one is most convenient for a certain purpose, we must exclude his work as the possible source of our structure.

Though it does not concern our argument immediately, the cosmological implication in Aristotle's words is worth noticing. Often it was taken for granted that the number of the 'wholeness of the heavens' is the same as that of the 'distance', twenty-four.[67] But what Aristotle says is: … ἐπὶ τὴν ὀξυτάτην νεάτην ἐν αὐλοῖς, ἧς ὁ ἀριθμὸς ἴσος τῇ οὐλομελείᾳ τοῦ οὐρανοῦ. Therefore an interpretation as twenty-four would be valid only if we might assume that the *bómbyx* is equated with one, or if we change the text. As it stands, the number of the heavens is that of the highest *nḗtē*, and so, according to our interpretation, thirty-two. It is hard to see, why thirty-two of all numbers should have been assigned such a metaphysical meaning. Much more plausibly in Aristotle's source the connection was made, not only between the musical scale, its numerical rendition, and the num-

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[67] Starting from Alexander of Aphrodisias, *in Metaph.* 814, p. 835.16–19 Hayduck. His 'explanation' (12 zodiacal signs, 8 spheres, 4 elements; obviously giving 24 as their sum and their least common multiple) is not meant to reproduce that of Aristotle's source, but is an invention of the author to illustrate how easy it is to construct impressive but meaningless numerical relationships. It is inspired by Aristotle's example, just as Alexander's preceding and following illustrations are inspired by Aristotle's examples of star numbers in constellations. Interesting is the final remark, ἀλλὰ καὶ οἱ ὀδόντες τῶν ἀνθρώπων τόσοι διὰ τὸν ἀριθμόν, which does not fit at all into the previous succession of examples introduced by ὅτι. Might it stem from an earlier commentary, which preserved traces of the equation of the οὐλομέλεια τῶν οὐρανῶν with the number 32? For the number of human teeth, cf. e.g. ps.-Hippocr., *Ep. Ptol. de hom. fab.* 287.23–289.1 (F.Z. Ermerins, Anecdota medica Graeca, Leiden 1840), Aristot., fr. 286 Rose; Gal., *UP* 3, 868 Kühn.

Nr.	Fixed Notes		Spheres
32	*nḗtē hyperbolaíōn*	a'	Fixed stars
24	*nḗtē diezeugménōn*	e'	Saturn
18	*paramésē*	b	Jupiter
16	*mésē*	a	Mars
12	*hypátē mésōn*	e	Sun Mercury Venus
9	*hypátē hypátōn*	B	Moon
8	*proslambanómenos*	A	Earth

Table 3: A reconstruction of the original fixed-notes cosmic harmony

ber of letters, but also to a version of the celestial harmony. In this case the 'wholeness of the heavens' can apply only to the outermost sphere of fixed stars. For any time in question, we can safely assume the knowledge of the seven planets.[68] With the fixed stars included, there are not enough scalar degrees for all planets. But we find Mercury and Venus united even in Ptolemy's version; and Plato already stresses that these planets are running together (namely at the same average speed[69]) with the Sun, with which they share their sphere in one of the 'ancient' systems mentioned by Plutarch. If we assume that the earth was already included, and set to *proslambanómenos*, as in all known variations of the fixed notes system, we can reasonably reconstruct an arrangement in which Sun, Mercury, and Venus shared the same note in the celestial concert, according to their equal speeds.[70] Such a tentative reconstruction is shown in Table 3.

Again, Philolaus can hardly be the source of such a list. In his work he obviously put forth the famous non-geocentric system of ten celestial bodies, including earth and counter-earth, which are reconcilable with our numbers only on the basis of implausible hypotheses.[71]

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68 Cf. Burkert 1962, 289–295.

69 In an epicyclical model, as probably accepted by Plato as a working hypothesis (cf. van der Waerden 1982), this implies an equal angular speed of the respective primary spheres.

70 It has bothered scholars that later systems assigned different notes to these three planets, despite the explicit Pythagorean connection of the planets' speed and pitch; cf. Aristot., *Cael.* 290b; Alex. Aphr., *in Metaph.* 31, p. 41.2–9 Hayduck; Burkert 1962, 314.

71 For instance, that earth and counter-earth shared the same speed just as the Sun, Mercury, and Venus do; or that the system incorporates a priori only the part of the universe visible to us. If Philolaus' sphere of fixed stars was not included as a moving body

A more likely alternative is Archytas, "the Pythagorean most concerned with music", according to Ptolemy.[72] His acoustical theory connects speed and pitch,[73] and so there is little doubt that his numerical relations were consistent with his physics, associating higher pitch with higher figures. Archytas makes it also explicit that astronomy, numbers, and music are related;[74] so his argument may well have culminated in a synoptic list. Moreover he was famous for his work on the different types of means, which play such an important role in the fixed notes framework. Plato used them to construct his *Timaeus* system,[75] and Ptolemy still counts all the instances of means occurring in the scale of his *Canobic Inscription*:[76] it seems that the idea of the means remained connected with the fixed notes variant of the cosmic scale. Unfortunately, from Archytas' work, only his account of the divisions of the tetrachord survives, not of the larger structures of music. However, if his progress in this field was only remotely as large as the step from Philolaus' tetrachord tunings to the refined Archytean system, we must certainly expect no less than the numerical representation of the Perfect System.[77]

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just as the central fire is not (cf. Huffman 1993, 254–257), there would not even be a number/note for the οὐλομελεία τῶν οὐρανῶν.

72 Ptol., *Harm.* 1.13, p. 30.9 f Düring.

73 Archyt., fr. 1, p. 433.13–435.10 D.-K.

74 Archyt., fr. 1, p. 432.4–8 D.-K.: περί τε δὴ τᾶς τῶν ἄστρων ταχυτᾶτος καὶ ἐπιτολᾶν καὶ δυσίων παρέδωκαν ἁμῖν σαφῆ διάγνωσιν καὶ περὶ γαμετρίας καὶ ἀριθμῶν καὶ σφαιρικᾶς καὶ οὐχ ἥκιστα περὶ μωσικᾶς. ταῦτα γὰρ τὰ μαθήματα δοκοῦντι ἦμεν ἀδελφεά.

75 One might wonder why Plato, who was on best terms with Archytas, did not use his diatonic tuning, but the older one. I suppose the reason must be sought in the closer connection of Archytas' tuning with musical practice, especially the irregular disjunction of *hyperypátē*, while Plato's cosmic harmony includes a maximum of concordant (in the ancient sense) intervals.

76 Ptol., *Inscr. Can.*, p. 154,11 f Heiberg.

77 Lydus, *Mens.* 18,48 Wuensch = Xenocrates, fr. 58 Heinze, quotes a numerical concept from Xenocrates to explain the connection of the moon with the number nine. The wording does not necessarily imply, as modern commentators seem to have taken it, that the latter idea is also found in Xenocrates (Heinze 1892, 70 f n. 3; Isnardi 1982, 380 f; but cf. the cautious diction of Huffman 1993, 372). Therefore the fragment does not support an origin of our cosmic harmony in the Early Academy or even earlier philosophy. If, on the other hand, the original context did already involve the moon, Xenocrates' curious argument (the number nine gives birth to itself) seems more apt for commenting on an already established numerological connection than to inspire it for the first time. After the planets had been identified with the numbers of the fixed notes sys-

Finally, we must remember that Aristotle refers explicitly to the aulos as the conceptual background of the numerical equation. This fits well into a picture of pre-Platonic Pythagoreanism, in which the string(s) on the canon had not yet taken over as a standard tool of relative pitch measurement,[78] and in which Archytas again seems to have played an important role:

> καὶ τῶν Πυθαγορικῶν δὲ πολλοὶ τὴν αὐλητικὴν ἤσκησαν, ὡς Εὐφράνωρ τε καὶ Ἀρχύτας Φιλόλαός τε ἄλλοι τε οὐκ ὀλίγοι. ὁ δ᾽ Εὐφράνωρ καὶ σύγγραμμα περὶ αὐλῶν κατέλιπεν· ὁμοίως δὲ καὶ ὁ Ἀρχύτας. (Ath. 184e, p. 402.20–24 Kaibel)
>
> And many of the Pythagoreans practised the art of the aulos, Euphranor, for instance, and Archytas, and Philolaus, and quite a number of others. Euphranor left even an essay on auloi; and Archytas did the same.

The importance of the aulos for pre-Aristoxenean music theory is further corroborated by the account Aristoxenus gives of the arrangements of *tónoi*, keys, as proposed by rivalling schools of theorists: one of these systems, he says, is oriented entirely towards the model of aulos boring, and another one sometimes expanded by adding 'the Hypophrygian aulos'.[79] Since the initial stages of the systematisation of *tónoi* were almost certainly independent of the conception of the Perfect System,[80] the aulos seems to

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tem according to the current knowledge of their order, a mind like Xenocrates' would feel the need to explain the ontological reasons behind those identifications.

[78] Of course, strings were used by some school(s) at the same time; cf. the famous mention of string-torturers in Plato, *Rep.* 531ab, who did, however, not carry out measurements; cf. West 1992, 240.

[79] Aristox., *Harm.* 2.37, p. 47.6–9 Da Rios: …ἕτεροι δὲ πρὸς τοῖς εἰρημένοις τὸν ὑποφρύγιον αὐλὸν προστιθέασιν ἐπὶ τὸ βαρύ. οἱ δὲ αὖ πρὸς τὴν τῶν αὐλῶν τρύπησιν βλέποντες τρεῖς μὲν τοὺς βαρυτάτους τρισὶ διέσεσιν ἀπ᾽ ἀλλήλων χωρίζουσιν… West 1992a, 31, deletes αὐλόν as an intrusion from αὐλῶν below, and indeed the text has suffered worse corruptions elsewhere. But as it is perfectly possible that there was such a thing as a 'Hypophrygian aulos', and that its existence (or rather invention?) triggered the extension of the key system, it is by no means necessary to change the text: cf. also Ath. 625a, p. 378.19–22 Kaibel, about the 'Hypodorian' originating in aulos design.

[80] Cf. the reconstruction in Hagel 2000, 165–182. Keys and Perfect System have perhaps merged not earlier than in Aristoxenus' *diágramma polýtropon*; and even there the full Perfect System was nowhere realized within a single key: cf. Hagel 2000, 183–188. It is only in the *sýstēma pentekaidekátropon* of the ancient notation that we find both concepts fully expressed.

have functioned as the model instrument for a wider range of theoretical concepts.

Aristoxenus himself, who usually pays little tribute to the achievements of his predecessors, acknowledges that 'those dealing with the musical instruments', unlike the rest of music theorists, had a clear conception of the interrelation of the genera, and thus of 'fixed' and 'moving' notes.[81] Accordingly, only these practice-oriented writers were in the position to develop the Perfect System – and the numerical sequence based on its fixed notes.

It is therefore plausible that the theoretical formulation of the Greater Perfect System went hand in hand with its practical realisation in aulos boring. Especially the presence of a *bómbyx* one octave below *mésē* seems of great use, whether in accompaniment or as a drone. Only this note stands in concordant intervals to the more important 'fixed' notes, just like *mésē* does: to *hyperypátē* and *hypátē* as well as to all three *nêtai*.[82] The origins of the Perfect System have usually been sought in a highly abstract effort of finding a grid for different lyre tunings or other types of *Gebrauchsskalen*;[83] a grid, however, which remains useless for all practical purposes. In a cultural context where philosophers did not only display great interest both for acoustics and for music theory, but, like everyone, were trained in singing, dancing, and, as we have seen, often also in aulos-playing, any explanation which presupposes a more intimate connection of music theory and practice must be regarded as a priori more plausible. The natural instrument of reference for the extension of scales as well as for heavily modulating systems was not the lyre with its slowly increasing but still limited number of strings,[84] but the versatile aulos, which was just then

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81 Aristox., *Harm.* 2.35 p.44.14–16 Da Rios.

82 For the interval of a tone and of a ninth to the remaining 'fixed' notes, *hypátē hypátōn* and *paramésē*, cf. the tone used in auletic music between accompaniment and melody as attested in ps.-Plut., *Mus.* 1137c (from Aristoxenos). See Winnington-Ingram 1928; West 1992, 206; Barker 1995, 50; Hagel 2004.

83 So e.g. West 1992, 223. The neat symmetric account of the system as given by Ptolemy has long obscured the independent origins of *tónoi* and 'octave species', which have at the same time often been mistaken for lyre tunings. The mechanism that ensures that any octave species lies roughly in the same pitch range if taken in its homonymous *tónos* is the necessary result of, not the reason behind the evolution of the system. See also n. 80 above.

84 When some theoreticians found it necessary to use many-stringed instruments (*psaltḗria* = harps or zithers?) to reproduce and examine contemporary music (Pap. Hi-

being equipped with newly invented mechanisms for closing and opening holes.[85] So the conclusions that we have based on the Aristotle passage rather corroborate a scenario which we should have expected anyway.

And what follows for the design of the aulos itself? As we have already seen, the *bómbyx* should usually have sounded the *proslambanómenos* (A) of the instrument's basic key, usable for a variety of concords, and perhaps as a drone,[86] an octave below the tonal centre, *mésē*. On the other hand, it must have been possible to play notes as high as *nḗtē hyperbolaíōn* (a'), Aristotle's 'highest *nḗtē*', represented by the highest number of our sequence, and located two octaves above the *bómbyx*. This cannot have been done without overblowing, just because finger holes for such high-pitched notes would lie very close to the mouthpiece, near the section usually occupied by the typical 'bulbs', and at the same time very close together.[87] The laws of physics lead us one step further: like any reed instrument with cylindrical bore throughout, the aulos must overblow to the twelfth.[88] If the lowest playable note is the *proslambanómenos* (A), then the first note available by overblowing will be *nḗtē diezeugménōn* (e'). Both notes can of course be accessed only if all finger holes are closed that lie below the lowest one which is, in the present playing position, actually fingered. This must have been done by the help of rotating sleeves or some other mechanism.[89] In other words, *nḗtē diezeugménōn* can be played without a finger hole of its own whenever the lowest (drone?) note is accessible. The even higher notes in the range of the *hyperbolaîon* tetrachord can be played on the holes at the lower end of the instrument. In this case, all the holes above the hand have to be closed mechanically. In terms of the Unmodulating Perfect System[90] there is no absolute need to have specific

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beh 13.23–31; cf. West 1992a, 16f; 21; Wallace 1995, 32–35), they testify to the impossibility of doing so on the kithara: obviously they investigate vocal and/or aulos music.

[85] Cf. West 1992, 87. On much earlier aulos-based music theory and the celebrated Athenian reaction against the instrument, cf. Wallace 2003.

[86] Cf. Byrne 2000.

[87] No Greek aulos fragment has such finger holes, nor is there any representation of respective fingering.

[88] The funny registers described by Najock (1996) are based not on a cylindrical bore, but one with sections of different diameter. Of the numerous aulos fragments, not one has such a design.

[89] On sleeves and sliders see Howard 1893, 7f; West 1992, 87f; Masaraki 1974; Litvinsky 1999; Byrne 2000.

[90] See Figure 1, above p. 59.

holes for any note above *nḗtē synēmménōn* (d'), because this note is also more or less identical in pitch with the second but highest note of the *diezeugménon* tetrachord in its highest tuning shade, *paranḗtē diezeugménōn diátonos*.[91]

In spite of the great number of excavated aulos fragments, almost none of them can serve for testing our conclusions. Most are either too fragmentary, or their restoration is doubtful, or they are of the more primitive type with only five finger holes plus a vent hole per pipe, or, in rare cases, from such a late and evolved type that an abundance of finger holes prevents the certain determination of a scale. And we almost never have the opportunity to study a pair of pipes that formed an instrument. The exception is the famous Louvre aulos: obviously a pair of pipes, one with nine, the other with seven finger holes, partially matching, partially complementing each other to form a sensible scale.[92] And this scale is identical with the predicted design: it ranges exactly from *proslambanómenos* to *nḗtē synēmménōn*. While the musically most important *mésē* (a) is the highest note present on both pipes, the discords with *mésē* and *bómbyx* are taken into consideration, too, as becomes clear from the treatment of the (necessarily discordant) neighbour notes of *mésē/proslambanómenos*. As regards the standing notes, the higher pipe stops short of *hypátē hypátōn* (B), the lower one of *paramésē* (b). Of the movable notes, on the other hand, it is precisely diatonic *likhanós* (g) that is bored to form concords (with c and d) only on the higher pipe, while the respective hole on the lower pipe, situated about a septimal tone below *mésē*, seems to have facilitated the production of certain shades of diatonic.[93]

Figure 3 displays the holes that need to be fingered to produce all notes of the scale, as discussed above:

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[91] Only in the highest shades of diatonic, the movable *paranḗtē* is about one tenth of a tone higher than the fixed *nḗtē synēmménōn*. This might have been taken into consideration in the Louvre aulos; see Hagel 2004. But pre-Aristoxenean music theory was concerned mainly with enharmonic music (cf. Aristox., *Harm.* 1,2 p. 6.6–12 Da Rios), and apparently 'diatonic' notes such as *paranḗtē diezeugménōn* were evidently understood as 'modulating' *nêtai* etc.; see Hagel (2000) 38–41. On the other hand, an additional higher hole would be needed for a (modulating) *pyknón* above *nḗtē synēmménōn* – which is, however, listed as *ápyknos* in Cleonid., *Harm.* 4, p. 186.5–7 Jan and Bacchius, *Harm.* 34, p. 300.6 f Jan.

[92] For a description of the instrument, see Bélis 1984. For the determination and discussion of its scale, Hagel 2004.

[93] See Hagel 2004, especially Table 2.

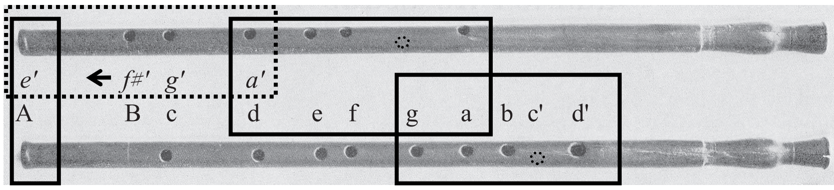

Figure 3: Playing the Louvre aulos

When the lower two finger holes are closed on the lower pipe, and perhaps also the lower four holes of the higher pipe, there are available, in addition to the *bómbyx*, all notes from *hyperypátē* (d) to *nḗtē diezeugménōn* (e'). This is exactly the famous central octave plus the tone below, which was most probably also the first extension of the old lyre tuning, taken at a rather early date. Without overblowing, the highest note is *nḗtē synēmménōn*, and the melodic compass (without the drone note) covers exactly one octave. It may be significant that this is the so-called Phrygian octave, whose practical relevance is well attested.[94] Its highest note could reasonably be called the 'Phrygian *nḗtē*', as opposed to *nḗtē diezeugménōn* (e') as the 'Dorian *nḗtē*',[95] associated more closely with the lyre. So it seems that just as 'Phrygian' music and the aulos are intimately connected, the 'Phrygian' mode was available on the aulos without the additional effort of overblowing.[96] The 'Dorian *nḗtē*' could be reached by simply overblowing the *bómbyx*, while each higher note of the *hyperbolaîon* tetrachord required the opening of a hole near the lower end of the tube, or moving the fingering position there entirely. In this case the melodic compass would shift from

94 In enharmonic shape it is described by Aristides Quintilianus (1.9, p. 18 f W.-I.) as belonging to the fifth century BC, in chromatic form in the cosmic scale known from several sources (Alexander of Ephesus in Theon, *Util. Math.* 138–141 Hiller; Censorinus, *de die natali* 13, p. 22.10–24.14 Sallmann; Ach. Tat., *Intr. Arat.* 17, p. 24 f Di Maria; with Dorian *nḗtē* in Pliny, *NH* 2.84), in diatonic form it seems to underlie the tonal structure of the Seikilos song, DAGM 23. The bounding notes are the same in all genera, and all notes can be produced on the Louvre aulos with the given fingering by partially covering holes to obtain the missing semitones and quartertones.

95 Ps.-Plut., *Mus.* 1140 f; cf. *Aristot. Pr.* 19.32, 920a; Plut., *Apophth. Lac.* 238 c.

96 By 'mode' I refer to music using such scales as transmitted by Aristides Quintilianus, separated from the notion of key: such music is indicated by the notation of those scales as well as of the Orestes fragment (DAGM 3) in the basic keys, which were (later) termed (Hypo)lydian. I plan to explain the history behind these conceptions in the near future. On mode in ancient Greek music, see Winnington-Ingram 1936.

d–e' to g–a', with an overlap of a sixth between both playing positions.[97] Of course there were other positions, too, which involved also the lower part of the higher pipe. The lowest four finger holes of this pipe can be fingered together, for instance, and all of them yield resonant intervals with the other pipe's *bómbyx*, from a minor third to a minor sixth.

Playing the *hyperbolaîon* tetrachord involves another complication. Since the aulos overblows not to the octave, but to the twelfth, the diatonic sequence of intervals of the higher register does not correspond to that one an octave below. As a result, a *hypátē hypátōn* finger hole produces not the desired *trítē hyperbolaíōn* (f), but a note that is one half tone too high (f#), corresponding roughly to *paranḗtē hyperbolaíōn khrōmatikḗ*. Accordingly, the correct pitch must be obtained by half-stopping the finger hole. But this was a common technique, necessary for many of the moving notes of the chromatic and enharmonic genera.

This, it seems, was the basic design of the standard professional aulos of about the early fourth century BC, and it presumably persisted as one possible design for centuries: the Louvre aulos is most probably from a later period.

If our conclusions are justified, they will affect another aspect of ancient Greek musical history. It has been observed that the *hyperbolaîon* tetrachord was not accounted for in the earlier stages of the ancient notation, and should therefore be relatively young.[98] But our interpretation of Aristotle's reference needs this tetrachord, and we have seen that it is integrated also into the Timaeus system. So I suggest that those early stages of the notation must be dated to the fifth century. It is not surprising that no direct technical reference to the tetrachord has survived from that time. Still the term is there, in a fragment from the comic poet Pherecrates. In one of his plays, Music herself complains about the violent treatment she has suffered from modern composers.[99] In the last lines, which unfortunately are not transmitted within their exact context,[100] she says:

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97 In the discussion following this paper, E. Pöhlmann drew a fascinating parallel to certain styles of Modern Greek traditional music, where there is also a change between two 'registers'. One could also compare the Delphic Paean by Athenaeus (DAGM 20), the melody of which usually moves between f and f', but at one place (l.4) temporarily drops to a low 'register' of c–f.

98 Cf. e.g. Jan 1894, 27.

99 Cf. Barker 1984, 237f. For an interpretation of the fragment in the light of extant ancient music, see Hagel 2000, 83–87.

100 The verses might immediately follow those cited before, so that ἐξαρμονίους

> …ἐξαρμονίους ὑπερβολαίους τ᾽ ἀνοσίους
> καὶ νιγλάρους… (ps.-Plut., *Mus.* 1142a)
>
> …inharmonious ones, and overshooting ones, impious,
> and shrill whistling…

No doubt already fifth century music knew something that was called *hyperbolaîos*, and that was new to such a degree that a conservative mind was ready to rebuke it as impious. Why should we assume that this feature was anything else than the high notes of the *hyperbolaîon* tetrachord?[101] Is it by chance that these are named in the same breath with whistling sound,[102] or may we take this as a reference to the diminished sound quality of those notes, which could be produced only by overblowing? At least it is hard to see why some sort of unpleasant whistling sound should have been introduced by those popular composers just for its own sake. If this is accepted, it becomes plausible that the term *hyperbolaîos* itself was coined in the context of aulos playing, and was originally associated with the increased effort that is necessary for the production of the second register. The entire Perfect System, ranging from the *bómbyx* to the 'overshoot' notes, appears then as a reflection of auletic practice.

Overblowing on the aulos is usually connected with the so-called *sýrinx*, an accessory sometimes mentioned in ancient sources, and tentatively identified as a speaker hole: it certainly served to facilitate the production of higher registers.[103] If our interpretation is correct, however, it is

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ὑπερβολαίους τ᾽ ἀνοσίους refers to χορδὰς δώδεκα, twelve strings/notes. But in this case νιγλάρους must be parallel to χορδάς: "undressed me and dressed me in twelve notes… and shrill whistling", which is rather odd. Moreover, it is hardly conceivable that the poet wanted to call all twelve notes 'overshooting'. So I suppose there is some text missing – if the passage is not from an altogether different play, by Aristophanes. Though the repeated use of χορδή might at first glance point to a context of stringed instruments, the passage as a whole is cited primarily as evidence for the decline of aulos-accompanied music; and χορδή has probably already its later meaning of 'note'.

101 Barker 1984, 238 n. 205 takes it as the natural interpretation.

102 The term ἐξαρμόνιος, on the other hand, obviously refers to modulations to alien keys (Hagel 2000, 85f), which probably originated also in auletic and aulos-accompanied music.

103 So first Howard 1893, 32–35; cf. Barker 1984, 226 n. 137. Becker's (1966, 68–80) arguments against the speaker theory are, like so much of his book, mostly based on untenable interpretations of the literary evidence. We need not be disturbed by the fact that perhaps none of the extant pipes and fragments actually has such a speaker hole.

not very likely that this *sýrinx* was already employed for overblowing the *bómbyx* to achieve the 'Dorian' *nḗtē diezeugménōn*, in what we might take as the standard playing position. Too often the player would have to switch the mechanism – which involved dragging a slide, not just pressing and releasing a button; so it must have been easier to accomplish the change of register just by means of embouchure. Especially if the transition did not involve a large interval, the overblown neighbouring note presumably came forth more readily than the deep *bómbyx* with its completely different oscillatory regime. But let us consider what Aristoxenus tells us, who is the only source that connects the *sýrinx* mechanism with the topic of instrument range:

> τάχα γὰρ ὁ τῶν παρθενίων αὐλῶν ὀξύτατος φθόγγος πρὸς τὸν τῶν ὑπερτελείων βαρύτατον μεῖζον ἂν ποιήσειε τοῦ εἰρημένου τρὶς διὰ πασῶν διάστημα, καὶ κατασπασθείσης γε τῆς σύριγγος ὁ τοῦ συρίττοντος ὀξύτατος πρὸς τὸν τοῦ αὐλοῦντος βαρύτατον μεῖζον ἂν ποιήσειε τοῦ ῥηθέντος διαστήματος.
>
> (Aristox., *Harm.* 1.20, p.26.8–27.3 Da Rios)

> For the highest note of the girls aulos would easily give an interval that exceeds the mentioned three octaves in relation to the deepest note of the oversize aulos; and if the *sýrinx* is drawn back, the highest note of the *sýrinx*-mode player would exceed the named interval in relation to the deepest note of the aulos-mode player.

If we take an aulos of the already large compass predicted by our interpretation and found on the Louvre instrument, we will reach exactly three octaves by overblowing the highest (*nḗtē synēmménōn*) hole to the first available harmonic, the twelfth – if it was possible to overblow such a high finger hole at all. Yet Aristoxenus talks about exceeding this interval, which will require either at least one finger hole more (which, as we have seen, is not very likely because of its necessary location close to the mouthpiece) or overblowing to the next available harmonic, the double octave plus third. On the other hand it is clear from Aristoxenus' wording that for him there is a much more pronounced difference between playing with the *sýrinx* (*syríttein*) and playing in normal mode (*auleîn*) than there is between different registers in a modern woodwind instrument. While on modern instruments the transition to overblown notes should cause as little modification of timbre as possible, ancient *syríttein* was obviously meant

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The chance of excavating exactly this type of virtuoso instrument, among so many other, and mostly simple, ones is not very good.

to imply not only a modification of pitch, but also of sound. Only so can we understand at all why a professional musician like Telephanes of Megara could reject the usage of the *sýrinx* mechanism altogether.[104] So this feature was obviously not used for extending the compass of melodic notes upwards: the *hyperbolaîon* tetrachord could be reached just by adjusting the embouchure.[105] The *sýrinx* will then have supported the production of the third register, in accordance with Aristoxenus' words: this register will cover a pitch range that extends from (roughly[106]) two octaves and a third up to three octaves and a sixth above the *bómbyx*, thus well exceeding a span of three octaves with its highest, but not its lowest note (cf. Figure 4). Accordingly none of the few ancient references indicates any connection with the aulos as accompanying vocal music. The *sýrinx* seems to have been a feature of professional solo performances, exemplarily associated with the mimetic representation of the dying Pythian dragon at the auletic contests during the festivals at Delphi.[107]

The aulos of our interpretation requires mechanical devices for opening and closing finger holes, such as rotating sleeves or sliders. Their invention is usually attributed to the famous Theban aulete Pronomus. At least he devised, we are told, and was the first to use, auloi on which it was possible to play in all three modes, Dorian, Phrygian, and Lydian – which had formerly required switching between three different instruments.[108] There is no doubt that such a modulating instrument did indeed require a kind of mechanism. Yet if Pronomus did not play his modes in different pitch ranges, his real invention might have been of an even more evolved kind: a type of sleeves that did not just open and close one hole, but al-

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104 Ps.-Plut., *Mus.* 1138a; Bélis 1999, 200–202. Howard's interpretation (1893, 33f: Telephanes displayed his ability to use the higher register in the old virtuoso way, without the help of a modern tool) fails to take into account the fact that he was not able to attend the Delphic games without a *sýrinx*.

105 For a skilled player it is possible to exert different pressure on the double reeds of two pipes, so that different registers can be used at the same time.

106 Since the archaeological evidence shows that the diameter of the finger holes did not vary according to their respective distances, the intervals of the overblown notes will not correspond exactly to those of the low register (cf. Benade 1960, 1593).

107 Cf. Strab. 9.3.10; see also n. 104 above. Solo playing is also implied in Xenophon, *Symp.* 6.5. Cf. West 1992, 102.

108 Paus. 9.12.5; Ath. 631e, p. 394.3–7 Kaibel. Bélis 1999, 200 seems to overlook Pausanias' explicit ἐπενόησεν.

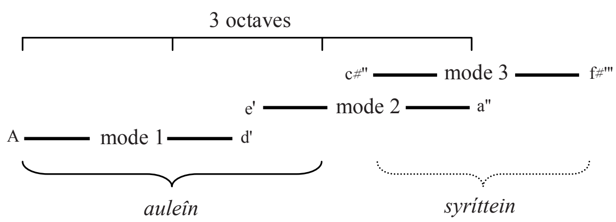

Figure 4: Aulos overblowing and the *sýrinx*

lowed one to alternate between two openings about a semitone apart.[109] On the other hand, if he devised such a modulating mechanism, we cannot know if it was already predated by a simpler type such as required by our considerations, or if this extension of range came later, or if both went hand in hand. In any case, the necessary mechanism must have been known well before the end of the fifth century. Its design involved a good bit of aulos-based investigation of musical scales, which further corroborates the rather early date of the theoretical efforts that ultimately led to the conception of the Perfect System.

So convergent evidence gives rise to a new picture of pre-Aristoxenian music theory. Much of it, and obviously the most concrete parts – the formulation of a scalar model as well as the arrangement of keys – was motivated by the practical needs of the ongoing evolution of aulos music, and presumably flourished in a discussion between theorizing professional musicians on the one hand and aulos-playing philosophers on the other. At the beginning of the fourth century, the Perfect system had obviously found its canonical form. Yet the vestiges of that exciting period have become largely obliterated. This might mostly be due to the 'Athenian reaction' against the aulos in general.[110] Though Aristoxenus was still an expert on aulos boring, too, almost all that was written on this subject fell into disregard once the instrument had assumed its ambivalent image in the mainstream of philosophizing literature. Accordingly, later theorists of the Pythagorean school concentrated on strings; anyway their mathematical demonstrations had now lost the connection with actual music.[111]

[109] Cf. Masaraki 1974.

[110] Cf. Wilson 1999; Wallace 2003.

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