QUASI THERMAL NOISE IN BERNSTEIN WAVES AT SATURN

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Abstract

On 1 July 2004, the Cassini spacecraft performed its Saturn orbit insertion (SOI), twice crossing the equatorial plane at ∼2.6Rs (from Saturn's spin axis) between the G and F rings. The HF radio receiver (HFR-RPWS) observed a peak at the upper-hybrid frequency f_{uh} and weak but steady banded emissions above the gyrofrequency, having well-defined minima at gyroharmonics. It was "*déjà vu*" with respect to the Earth and Jupiter magnetospheres with spacecraft using similar instrumentation. Since SOI, we have observed such features around most of the Cassini perikrones (periapses at Saturn), sometimes including emissions between the gyroharmonic bands above f_{uh}. We interpret these noise bands between gyroharmonics as quasi-thermal noise in Bernstein modes, which are electrostatic waves propagating almost perpendicularly to the magnetic field. The weak steady emissions between the gyroharmonics are mainly sustained by the suprathermal electron population and may be enhanced above f_{uh} by specific resonances of Bernstein modes – the so called f_Q frequencies. Their observation depends on the antenna shape and length compared to the basic plasma scales and requires a receiver with adequate sensitivity. On Cassini, we show that they can be detected below f_{uh} whenever the local electron gyroradius is shorter than ∼9m.

1 Introduction

From the first explorations of planetary magnetospheres, it is well known that the spectral density measured with an electric antenna peaks at the upper hybrid frequency

$$f_{uh} = \sqrt{f_g^2 + f_p^2},$$

where $f_g = \omega_g/2\pi$ is the electron gyrofrequency (or cyclotron frequency) and $f_p = \omega_p/2\pi$ is the plasma frequency. It was observed early in the inner magnetosphere of Earth

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[e.g. Gregory, 1971] and then at the giant planets [e.g. Birmingham et al, Gurnett et al., 1981]. The instrumentation, however, only allowed the detection of strong peaks at specific plasma resonances or strongly growing plasma instabilities, but did not enable the detection of weak steady emissions above the background noise. More recently, the Ulysses spacecraft crossed the Io plasma torus and the WIND spacecraft went through the Earth plasmasphere, both carrying spectrometers clean and sensitive enough to measure weakly banded emissions above f_g as shown on the two left panels of Figure 1. On the two right panels of this figure, one can see recent dynamic spectra acquired by the RPWS-Kronos receiver using the electric dipole antenna of Cassini, during the Cassini SOI on 1st July 2004 (top right), and near the perikrone of orbit 11 on 14-15 July 2005 (bottom right). On these four spectrograms, one can observe very similar spectral features: either, when $f_g \ll f_p$, a peak at f_{uh} with a weak power enhancement between the harmonics of the gyrofrequency (except sporadic instabilities, mainly present in the first gyroharmonic band), or, when f_p is only a few times larger than f_g, a relatively strong power enhancement between the gyroharmonics (this arises after 18 UT on the Ulysses dynamic spectrum or after 0 UT on the Cassini spectrum of orbit 11).

Our interpretation is that these steady and weak banded emissions do not result from instabilities, but instead are the quasi-thermal noise that can be calculated from the classical theory of plasma fluctuations in the presence of a strong magnetic field, including contributions of electrostatic waves propagating almost perpendicularly to the magnetic field, the Bernstein modes. From this interpretation ((detailed in section 2), we explain in section 3 how we can predict when we can detect (or not) the Bernstein waves below f_{uh} with Cassini's electric antennas in the inner magnetosphere of Saturn.

2 Basics of Quasi Thermal Noise in Bernstein Modes

In the absence of instabilities, the spectral density measured at the terminals of an electric antenna immersed in a plasma is essentially [1] the power spectrum of the potential induced by the thermal motion (or quasi-thermal, if the velocity distribution is not a Maxwellian) of the ambient electrons and ions – the quasi-thermal noise (QTN hereinafter). This voltage power spectrum V_ω^2 can be written :

$$V_\omega^2 \;=\; \frac{2}{(2\pi)^3}\int d^3k\,|\mathbf{J}(\mathbf{k})|^2\,E^2(\mathbf{k},\omega) \tag{1}$$

that is formally a 3-dimensional integration over all electrostatic waves of wave vector $\mathbf{k}$ contributing to the spectral density. Here $\mathbf{J}$ is the Fourier transform of the current distribution on the antenna (the "antenna response term") and E^2 is the autocorrelation spectrum of the ambient electric field which depends on the plasma parameters via :

$$E^2(\mathbf{k},\omega) = 2\pi\frac{\sum_j q_j^2 \int d^3v \quad f_j(\mathbf{v})\,\delta(\omega - \mathbf{k}\cdot\mathbf{v})}{k^2\varepsilon_0^2\,|\varepsilon(\mathbf{k},\omega)|^2} \tag{2}$$

[1] the QTN of electrons dominates between f_g and f_{uh}, while the shot or impact noise, due to particles which directly impact the antenna body (or the spacecraft body for the monopole antenna) is dominating in the lowest frequency band of the spectrum.

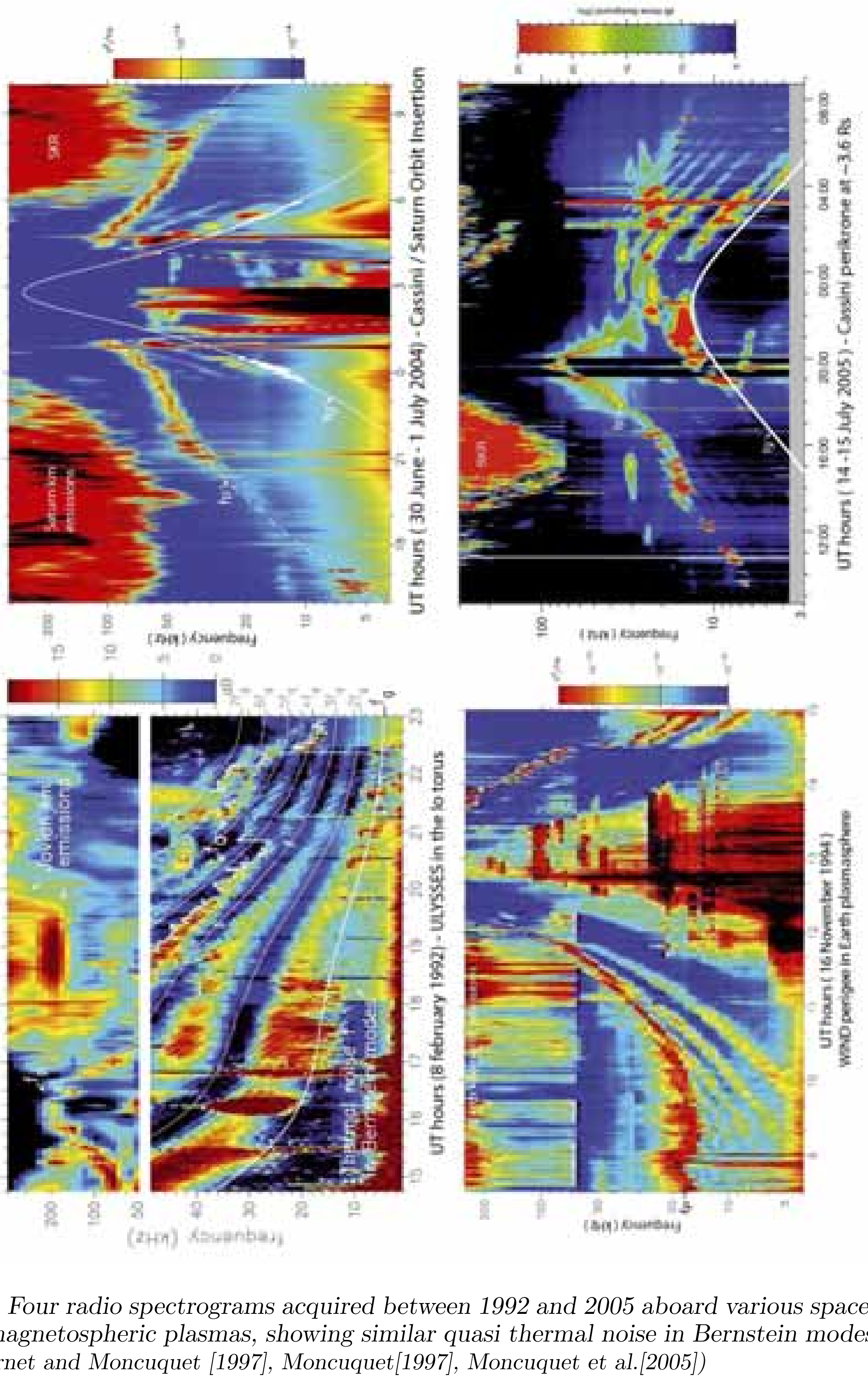

Figure 1: Four radio spectrograms acquired between 1992 and 2005 aboard various spacecraft in various magnetospheric plasmas, showing similar quasi thermal noise in Bernstein modes. (from Meyer-Vernet and Moncuquet [1997], Moncuquet[1997], Moncuquet et al.[2005])

where q_j is the charge, f_j is the velocity distribution of the jth particle species, δ a Dirac function, and ε is the plasma dielectric function, which depends itself on the velocity distribution and on the (possibly strong) ambient magnetic field.

We consider here frequencies sufficiently high that ion motions and impact noise can be neglected and shall assume the electron velocity distribution $f(v)$ in the inner planetary magnetospheres to be a sum of two Maxwellians (core+halo) of total density $N_e = N_c + N_h$. T_c and T_h are the temperatures for the core and halo population respectively. The calculation of the QTN with such a core+halo distribution has been done by Meyer-Vernet and Perche [1989] for a non-magnetized plasma and extended to the magnetized case by taking into account the contribution of Bernstein waves by Sentman [1982] and Meyer-Vernet et al. [1993], under the assumption that the plasma is stable. Let us summarize an important point of the QTN calculation: for a non-magnetized plasma, the main contribution to the integral (1) comes from waves with wavelength greater than or close to the core electron Debye length $L_D = \sqrt{k_B T_c/m_e}/\omega_p$. For a magnetized plasma, additional QTN comes from Bernstein waves with wavelengths close to the core electron gyroradius (or Larmor radius) $\rho_c = \sqrt{k_B T_c/m_e}/\omega_g$ (if $\rho_c > L_D$).

More precisely, Bernstein waves are an infinite set of undamped longitudinal modes propagating perpendicularly to $\mathbf{B}$ (i.e. $k_\parallel = 0$) between harmonics of the gyrofrequency. The dispersion equation for Bernstein waves has the implicit form:

$$\varepsilon = 1 - 2 \sum_{c,h} \frac{\omega^2_{p(c,h)}}{\omega_g^2} \frac{e^{-k_\perp^2 \rho_{c,h}^2}}{k_\perp^2 \rho_{c,h}^2} \sum_{n=1}^{+\infty} \frac{n^2 I_n(k_\perp^2 \rho_{c,h}^2)}{\omega^2/\omega_g^2 - n^2} = 0 \qquad (3)$$

where I_n is a modified Bessel function of the first kind and $k_\perp$ is the wave vector component perpendicular to $\mathbf{B}$. The solutions of the Bernstein wave dispersion equation (3) are shown e.g. in Moncuquet et al. [1997] . Note that for oblique propagation, the dispersion equation has an imaginary part, so that the waves are damped; however if $|k_\parallel| \ll k_\perp$ they remain rather similar to Bernstein modes. For parallel propagation ($k_\perp = 0$), the dispersion equation reduces to that without a magnetic field. Note this is valid for low-β plasmas, so that transverse and longitudinal modes decouple (we have typically $\beta < 5.10^{-3}$ along the Cassini orbits studied here). We may thus summarize the basics for calculating the QTN in a magnetized plasma: when restricted to parallel propagation, the integration of Eq.(1) yields roughly the same result as without a magnetic field, and the damping of oblique propagation excludes any additional wave contribution to the QTN except for vanishing $k_\parallel$, that is Bernstein modes.

In order to estimate the contribution of Bernstein modes to QTN at a given frequency, we must distinguish between two cases, depending on whether this frequency is located above or below the upper-hybrid band :

1. The frequency belongs to an intraharmonic band below f_{uh} where there is a unique solution of (3): in this case the Bernstein modes will increase the QTN between the gyroharmonics to a maximum level which mainly depends (out of instabilities) on the suprathermal electrons (the halo). Let us briefly explain this point: a general condition for electrons of velocity $\mathbf{v}$ to resonate with an electrostatic wave $(\mathbf{k}, \omega)$ in a magnetized plasma is $\omega - k_\parallel v_\parallel = n\omega_g$, where n is an integer. So "true" Bernstein modes ($k_\parallel = 0$) are damped at gyroharmonics but propagate without damping between them. Small $k_\parallel$ modes between the gyroharmonics resonate with the halo (because this requires large $v_\parallel$), and at the gyroharmonics with the core (small $v_\parallel$).

Thus, Bernstein modes enhance the QTN above the thermal (core) level only between the gyroharmonics. The computation of the QTN in this case has been done by Moncuquet et al.[2005] who deduced the temperature T_h of the halo electron population encountered along the SOI trajectory of Cassini; we make this computation explicitly in the following section to set up a general condition for the detection of Bernstein waves below f_{uh} with the Cassini antennas.

2. The frequency belongs to the f_{uh} band or to an upper intraharmonic band, where there exist solutions with zero group velocity – the so called f_Q resonances : in that case, the resonance will strongly enhance the QTN at this f_Q frequency. Note there is also a forbidden band (where no Bernstein waves propagate) between f_Q and the next upper gyroharmonic band in which the spectral density plummets – see Moncuquet et al. [1997]; unfortunately, the insufficient spectral resolution of RPWS (48 log-spaced channels from ∼3 to 300 kHz) does not allow their detection. The calculation of the QTN level at f_Q is beyond the scope of this paper (see e.g. Sentman [1982]); in practice these f_Q resonances might be observed when f_p is of the order of or a few times f_g, that is, on Figure 1, after 18 UT on the Ulysses spectrogram (top left) or after 0 UT on the Cassini spectrogram of orbit 11 (lower right). On all other parts of the spectrograms of Figure 1 where Bernstein waves are detected, we have $f_g \ll f_p$, which belongs to case 1.

Let us finally note that minima of QTN will always occur at harmonics of the gyrofrequency, where the contribution of Bernstein modes tends to zero. We could thus exploit this fact on every perikrone of Cassini where Bernstein modes are detected in order to determine f_g. During SOI, we derived f_g in this way with an accuracy of $\sim 2\%$ when compared to the magnetometer (MAG) experiment (see Figure 3 of Moncuquet et al.[2005] and the white curves plotted on the spectrograms of Cassini SOI and of Ulysses in Figure 1, which have been determined the same way).

3 QTN in Bernstein Modes below the upper-hybrid frequency

Let us now estimate the noise where it peaks below f_{uh}, that is roughly midway in each intraharmonic band: we may derive from the dispersion equation (3) the following estimation of the QTN between the gyroharmonics (adapted from Eq.22 of Meyer-Vernet et al.[1993]) :

$$V_\omega^2 \approx \frac{F_\perp^V(k_\perp L\eta)}{\pi\epsilon_0/\sqrt{k_B m_e}} \left[\frac{(\omega_g/\omega)\Delta k_\parallel/k_\perp}{|\partial\epsilon_r/\partial k_\perp\rho_c|_{k_\parallel=0}}\right] \frac{T_h}{\sqrt{T_c}} \tag{4}$$

Here $F_\perp^V$ is the response of a V-shaped wire antenna (of V-angle Φ) to Bernstein modes, $\eta = \max(\cos\theta\cos\Phi/2, \sin\theta\sin\Phi/2)$, where θ is the angle between the Cassini X-axis of the dipole antenna and $\mathbf{B}$, ϵ_r is the real part of the plasma dielectric function, $k_\perp$ is the unique solution (for $\omega < \omega_{uh}$) of the Bernstein dispersion equation, $k_\parallel$ is a small parallel component such that $|k_\parallel| \leq \Delta k_\parallel \ll k_\perp$ of waves excited between the gyroharmonics by hot electrons (see point 1 of section 2).

Note first that the bracket in (4) is weakly dependent on the plasma parameters (see Meyer-Vernet et al. [1993]); for the purpose of the present study we only need to know that this term is nearly constant along the parts of Cassini orbits considered. Midway in the intraharmonic bands $[2f_g, 3f_g]$ or $[3f_g, 4f_g]$ (the most frequently observed cases with Cassini), we have $k_\perp \rho_c \approx 2$ (see the dispersion curves in Moncuquet et al. [1997]), so that $k_\perp L\eta \sim 2\eta L/\rho_c$. The voltage power level from equation (4), near the 2nd or 3rd mid-gyroharmonic band, may thus be approximated by:

$$V_{mid}^2 \;\propto\; F_\perp^V(2\eta L/\rho_c)\; T_h/\sqrt{T_c} \tag{5}$$

The measured voltage, therefore, depends strongly on the antenna response $F_\perp^V$, which itself depends on two parameters: the inclination of the antenna to the magnetic field, which determines η, and the antenna length compared to the electron Larmor radius. Consider first the parameter η: it plays an important role for a spinning spacecraft (as Ulysses) because it provides a strong modulation of the signal; this was exploited on Ulysses by Meyer-Vernet et al.[1993] and Moncuquet et al.[1995] to provide accurate measurements of the electron temperature in the Io plasma torus. On Cassini, which is 3-axis stabilized, η remains relatively constant around the perikrones, with $\eta \sim 0.5$ during all the studied periods of SOI, except during a Cassini operation at ∼5:20 UT when it briefly reaches 0.9, producing, indeed, an enhancement of the QTN between gyroharmonics (not shown).

We now consider the antenna response behavior $F_\perp^V(L/\rho_c)$: how does it control the Bernstein mode contribution to QTN ? When adapting the computation of $F_\perp$ from Moncuquet et al.[1995] to a V-shaped dipole, it yields approximately: $F_\perp^V(u) \simeq [8/us][2\int_0^u dt J_0(t) - 2J_1(u) + J_1(2us)/s - \int_0^{2us} dt J_0(t)/s]$, where $s = \sin\Phi/2$ and $J_{0,1}$ denotes the Bessel functions of the first kind. Then, using $F_\perp^V(u) \sim (su)^2$ and $\sim 8/u$ for small and large u respectively, we obtain:

$$V_{mid}^2 \propto (sL/\rho_c)^2 \;\; (T_h/\sqrt{T_c}), \qquad \text{for} \quad \rho_c \lesssim L \tag{6}$$

$$V_{mid}^2 \propto T_h/(Lf_g), \qquad \text{for} \quad \rho_c \ll L \tag{7}$$

Eq.(7) shows that the level of QTN at midband is merely proportional to the temperature of the halo for small ρ_c, so mainly when the magnetic field is relatively strong. Eq.(6) shows that the signal enhancement between gyroharmonics vanishes rapidly for $\rho_c \gtrsim sL$ (this is nothing but the fact that the antenna is mainly sensitive to wavelengths of the order of its own length). So for a receiver sensitive enough, the Bernstein modes may be detected as soon as the local gyroradius becomes equal to or smaller than sL. The dipole of Cassini is a V-shaped antenna with each arm of length $L = 10$ m forming an angle $\Phi = 120°$, whose effective length for Bernstein wave detection is ≈ 8.7m. It is actually the limit of Bernstein mode detection we achieved during the SOI, as shown in Fig.2 where we have plotted ρ_c from our f_g and T_c determinations, i.e. when the Bernstein modes are effectively observed. Let us finally remark that the detection or not of the noise peak at f_{uh} is a problem similar to the detection of Bernstein waves. At $\sim f_{uh}$, the power spectrum exhibits a maximum which behaves much like the peak near f_p in an isotropic plasma ($f_g \ll f_p$). The peak level near f_p has been estimated by Meyer-Vernet and Perche[1989], who showed that the peak vanishes rapidly for $L_D \gtrsim L/2 = 5$m. We may verify this point

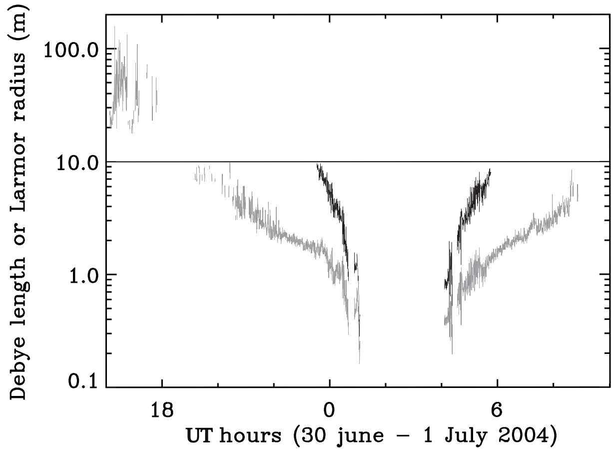

Figure 2: Local Debye length (grey) or Larmor radius (black) along the Cassini SOI trajectory.

on Fig.2, where the L_D obtained from our QTN measurements practically do not exceed 5 m, except for the measurements before 18 UT, which were not acquired from the QTN, but this is another story…

4 Conclusion

The spectral density measured on 1 July 2004 with the Cassini antennas in the inner magnetosphere of Saturn was qualitatively and quantitatively the "QTN in Bernstein modes" that we could expect (see Moncuquet[1997] for a qualitative prediction) by adapting the QTN spectroscopy we have successfully performed [Meyer-Vernet et al., 1993, Moncuquet et al., 1995, 1997] on the spectra acquired by Ulysses on 8 February 1992 in the Io plasma torus (IPT). It is also noteworthy that the spectrogram acquired on 14-15 July 2005 by Cassini shows properties very similar to the QTN in Bernstein modes found in the IPT, although with very different antennas immersed in a less dense and colder plasma.

We have shown evidence that Bernstein modes are detected below f_{uh} as soon as the local electron gyroradius becomes shorter than the effective length of the electric antenna, just as the plasma frequency can be detected when the local Debye length is shorter than the antenna length. Above f_{uh} some resonances of Bernstein modes – the so called f_Q – can also be detected when f_p is only a few times larger than f_g (as shown on both Ulysses and Cassini spectrograms of Figure 1).

From a pure 'space physics' point of view, it is noteworthy that the Bernstein modes, often called "electron cyclotron emissions" or, more empirically, "n+1/2" emissions, are ubiquitous in planetary magnetospheres, and can be interpreted simply as steady quasi-thermal noise sustained by the suprathermal electron population or due to f_Q resonances. They can be observed or not, depending on the adequateness of the antenna design to the plasma environment and the receiver sensitivity. These electrostatic fluctuations can sometimes be excited by plasma instabilities, but, in any cases, they are strongly damped at the harmonics of the local electron gyrofrequency, as the theory predicts. This last point may allow an accurate measurement of the **B** magnitude near the Cassini perikrones, when Bernstein modes are detected. We also plan to deduce from this QTN spectroscopy in Bernstein modes the major physical parameters of the Kronian plasma torus around each Cassini perikrone, as has been done by Moncuquet et al. [2005] at the Saturn orbit insertion.

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