PROPAGATION OF ALFVÉN WAVES GENERATED BY THE INTERACTION OF TITAN WITH MAGNETOSPHERIC PLASMA

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Abstract

This paper deals with the propagation of Alfvén waves along the dipole magnetic field lines from Titan towards the ionosphere of Saturn. Numerical simulations show that the length scale of Alfvén perturbations propagating along the converging magnetic field lines increases proportional to the magnetic field strength. A strong convergence of magnetic field lines leads to reflection of the Alfvén pulse from a very narrow part of the magnetic tube. Because of this effect, the Alfvén wave pulses have many reflections from the ionosphere without a noticeable damping. Two cases, meridional and azimuthal Alfvén wave polarizations, are examined. For the first case, the wave energy damping is shown to be stronger than that for the second case.

1 Introduction

The interaction of Titan with the ambient plasma leads to the draping of the magnetic field lines and strong disturbances of plasma environment. Numerical simulations indicate the appearance of Alfvén wings downstream of Titan [Ledvina et al., 2004], which are expected to evolve asymptotically to Alfvén wave pulses propagating far away along the converging magnetic field lines towards the ionosphere of Saturn.

In the general case, the problem of the propagation of magnetohydrodynamic waves in a nonuniform magnetic field is complicated to solve. With regard to the Earth's magnetosphere, a fruitful simplifying assumption of large azimuthal wave numbers was proposed (see [Klimushkin, 1998], and references therein), which imply the longitudinal wave length to be much larger than the azimuthal one. In this approach, the fast magnetosonic mode is strongly evanescent, and only transverse Alfvén waves coupled to slow mode magnetosonic waves can be described. In such a case the perturbation of the total pressure is

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zero. This assumption was used also in the magnetic string approach by Erkaev et al. [2005].

The aim of this paper is to analyze the propagations of Alfvén wave pulses along a dipole-like magnetic tube from Titan towards Saturn and their reflections from the conducting ionosphere. In particular, the effects related to the convergence of magnetic field lines are studied.

2 Statement of problem

In the dissipationless approximation, the magnetic field and plasma parameters are determined by the ideal MHD equations which are commonly used for space plasmas,

$$\rho\frac{\partial \mathbf{V}}{\partial t}+\rho(\mathbf{V}\cdot\nabla)\mathbf{V}+\nabla\left(P+\frac{1}{8\pi}B^2\right)-\frac{1}{4\pi}(\mathbf{B}\cdot\nabla)\mathbf{B}=0, \tag{1}$$

$$\frac{\partial\rho}{\partial t}+\operatorname{div}(\rho\mathbf{V})=0,\quad \frac{\partial}{\partial t}\left(\frac{P}{\rho^\gamma}\right)+(\mathbf{V}\cdot\nabla)\left(\frac{P}{\rho^\gamma}\right)=0, \tag{2}$$

$$\frac{\partial \mathbf{B}}{\partial t}-\operatorname{rot}(\mathbf{V}\times\mathbf{B})=0,\quad \operatorname{div}\mathbf{B}=0. \tag{3}$$

Here, ρ, $\mathbf{V}$, P, $\mathbf{B}$ are the mass density, velocity, plasma pressure, and magnetic field, respectively, and γ is the polytropic index. In our simulations we use $\gamma=2$ for simplicity. In fact, our work is focussed on the Alfvén waves which are not sensitive to a polytropic index value.

The magnetic field lines are assumed to be connected with the ionosphere of Saturn which has a finite conductivity. At the boundary surface, we use the relationship between the electric field and magnetic field perturbations, $\mathbf{n}\times\delta\mathbf{B}\,c/(4\pi)=\Sigma_s\mathbf{E}$, which is based on a local Ohm's law. Here, Σ_s is a conductivity of the boundary surface, and $\mathbf{n}$ is the normal unit vector. The undisturbed magnetic vector is assumed to be parallel to the unit vector $\mathbf{n}$. For an ideal conducting plasma in the magnetic tube, the electric field is $\mathbf{E}=-\frac{1}{c}\mathbf{V}\times\mathbf{B}$, where c is the speed of light, and $\mathbf{V}$ is the plasma velocity. After substituting the electric field, the boundary condition is $\mathbf{V}=\mu\delta\mathbf{B}/\sqrt{4\pi\rho}$, where $\mu=c^2/(4\pi V_{as}\Sigma_s)$, and V_{as} is the local Alfvén speed. This boundary condition is consistent with that derived by Leonovich and Mazur [1990]. The quantity μ is not well known. Dissipation effects for reflecting waves vanish for $\mu=0$, and they are maximal for $\mu=1$. In our numerical calculations we take an intermediate value, $\mu=0.5$.

A dipole magnetic field configuration is considered (see Figure 1). At the equator, another boundary condition is given for the plasma velocity perturbation which is varying between positive and negative values during the initial short period, and after that, it is assumed to vanish. The length scale a of the wave perturbation initiated at the equatorial boundary is chosen to be of order of the diameter of the Titan's ionosphere, $a\sim 7000$ km. This is the minimal estimation of the wave length scale. In such a case, the duration of the wave pulse is equal to a/V_{a0}.

Considering the Alfvén waves to be the most important, we study the propagation of the Alfvén pulses and analyze the effects related to convergence of the dipole magnetic

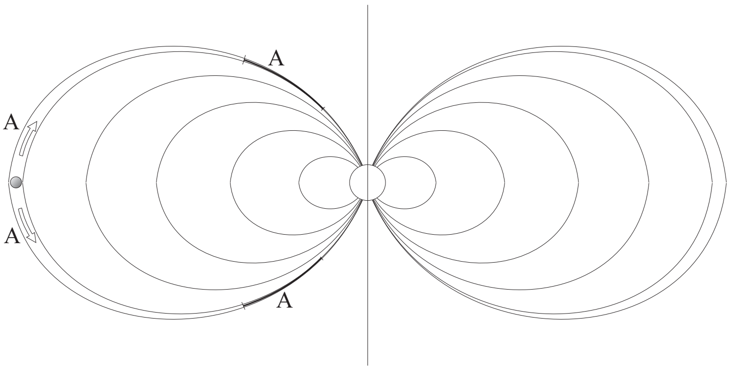

Figure 1: Geometrical situation of the problem. The Alfvén pulse (A) propagating along the dipole magnetic tube is indicated at the initial position, and at the reflection zone.

field lines. For large plasma betas, the group velocity of slow modes deviates from the magnetic field, and therefore the slow mode perturbations are not confined within the magnetic tube. In this case (relevant to the Titan's orbit), the slow mode amplitude is damped rather quickly [Langmayr et al., 2004]. It follows from Erkaev et al. [2005] that the coupling between the Alfven and slow mode waves is determined by the parameter $\beta\lambda/R$, where R is a curvature radius, and λ is a length scale of the wave perturbation ($\lambda \sim 7000$ km for Titan). This parameter is small for the case of waves produced by Titan.

3 Results of calculations

The method to solve this problem is described by Erkaev et al. [2005]. It is based on the magnetic string approach with material "frozen-in" coordinates and a finite difference Lax-Wendroff scheme. For calculations of wave propagations we use a linearized form of MHD equations with respect to perturbations of magnetic field and plasma parameters.

The distance ($\mathbf{r}$), time (t), mass density (ρ), pressure (P), Alfvén wave perturbations of the magnetic field ($\delta\mathbf{B}$) and the velocity ($\mathbf{V}$) are normalized as follows,

$$\tilde{\mathbf{r}} = \frac{\mathbf{r}}{R_0}, \quad \tilde{t} = t\frac{V_{a0}}{R_0}, \quad \tilde{\rho} = \frac{\rho}{\rho_0}, \quad \tilde{P} = P\frac{4\pi}{B_0^2}, \quad \delta\tilde{\mathbf{B}} = \frac{\delta\mathbf{B}}{\delta B_0}, \quad \tilde{\mathbf{V}} = \frac{\mathbf{V}}{\delta B_0}\frac{B_0}{V_{a0}}, \tag{4}$$

where R_0 is the radius of Titan's orbit, V_{a0} is the Alfvén speed, ρ_0 and B_0 are the mass density and magnetic field strength refered to the Titan's orbit, δB_0 is the initial amplitude of the magnetic field perturbation. The plasma parameters and normalization quantities corresponding to the orbit of Titan [Neubauer et al., 1984] are given in Table 1. Here R_s is the radius of Saturn, and C_{s0} is the sonic speed. The total mass density is determined as $\rho_0 = m_{N+}n_{N+} + m_{H+}n_{H+}$, where m_{H+} and m_{N+} are the masses of hydrogen and nitrogen ions, respectively, while n_{H+} and n_{N+} are the number densities of the corresponding ions. The dipole magnetic field magnitude has been used in our calculations of the Alfvén wave propagations. The mass density is assumed to be constant. We justify this in the following way. In equilibrium, the distribution of plasma density along the magnetic tube for each

Table 1: Plasma parameters and normalization quantities

n_{H+} (cm^{-3})	n_{N+} (cm^{-3})	ρ_0 (g cm^{-3})	R_0 (R_s)	V_{a0} (km/s)	B_0 (nT)	C_{s0} (km/s)	R_0/V_{a0} (hours)
0.1	0.2	4.85 10^{-24}	20	64	5	210	5.22

ion specie is determined by the Boltzmann law: $\rho_i = \rho_{i0}\exp(m_i\Phi/(kT))$, where Φ is a sum of gravitation and centrifugal potentials, $\Phi = GM_s(1/r - 1/R_0) + 1/2\,\Omega^2[r^2\sin(\theta)^2 - R_0^2]$. Here, subscript i denotes the species of ions, ρ_{0i} is the mass density at the equator, r is a spherical radius, Ω is the angular speed of the magnetic tube corresponding to Titan (which is less than the angular speed of Saturn), and θ is the angle from the rotational axis. The Alfvén velocity is dependent on the square root of plasma density as $B/\sqrt{4\pi\rho}$. For the plasma parameters of Titan's orbit (two species) we found a variation of $\sqrt{\rho}$ about 15% in a range of distances $10R_s < r < 20R_s$. Considering this variation of mass density as to be insignificant (as the first step), we have done our calculations of the Alfvén wave propagations just for constant mass density.

Figures 2, 3, 4 present the velocity, magnetic field, and wave energy profiles corresponding to the Alfvén wave pulse propagating along the dipole magnetic flux tube in the case of meridional polarization.

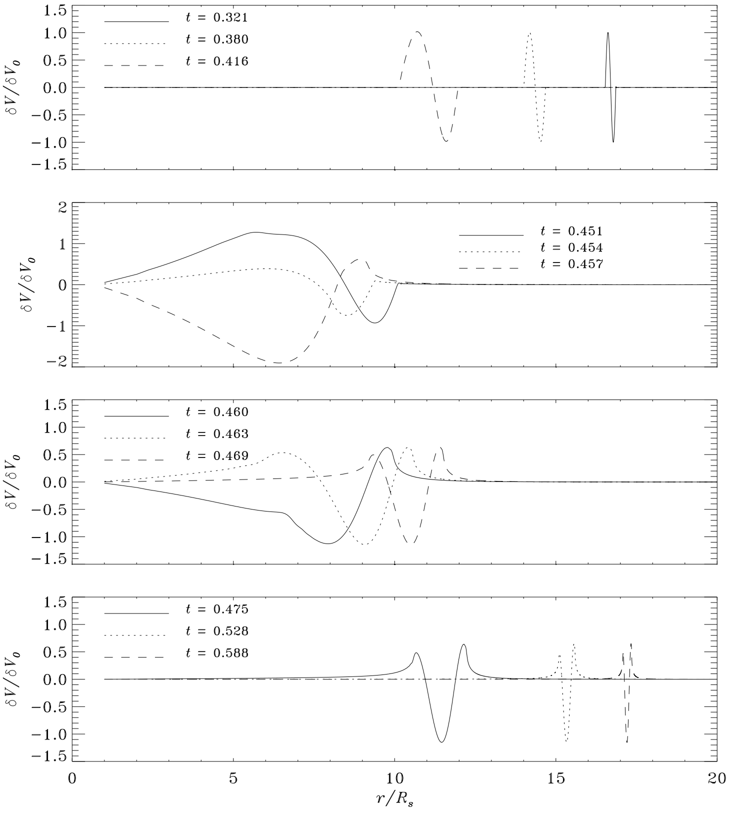

Figure 2: Perturbation of velocity normalized to $V_0 = \delta B_0 V_{a0}/B_0$ as a function of radial distance for different times.

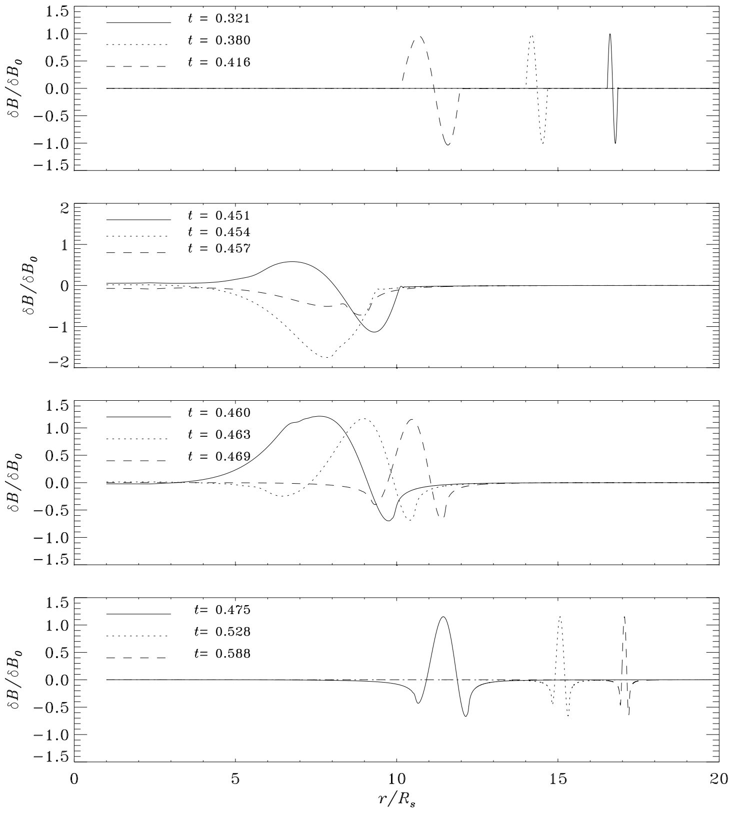

Figure 3: Perturbation of magnetic field as a function of radial distance for different times

Figure 2 shows the velocity perturbations distributed along the magnetic tube for different times. Initially, the pulse is rather short, and its length increases very much while the pulse is propagating towards Saturn. The reflection starts at the dimensionless time $t^* = 0.451$. Multiplying this number with the scaling factor, we find the propagation time to be 2.356 hours. During this time, the Alfvén pulse goes from the equator to the reflection zone. In the reflection zone, the length of the pulse is of order of 10 R_s. The shape of the reflected pulse is very different from the initial one.

Figure 3 shows the magnetic field perturbations in the magnetic tube. Initially they have the same polarity as the velocity. But after reflections the magnetic field perturbations have opposite polarity compared to that of the velocity.

Figure 4 indicates the behavior of the energy flux density along the magnetic tube for different times. The energy flux is positive for the pulses propagating towards Saturn, and it is negative for reflected pulses. The change of sign of the energy flux at $t^* = 0.451$ is the evidence of the wave reflection. The reflected Alfvén pulse goes back to the other hemisphere where it will be reflected again. After that, it will return back to the initial point at the equator. The total period of this propagation can be estimated as $\tau = 4t^* R_0/V_{a0} = 9.424$ hours.

Figure 5 show the total wave energy as a function of time given in units of the double propagation time from the equator to the reflection zone, $2t^* R_0/V_{a0}$. The energy decrease

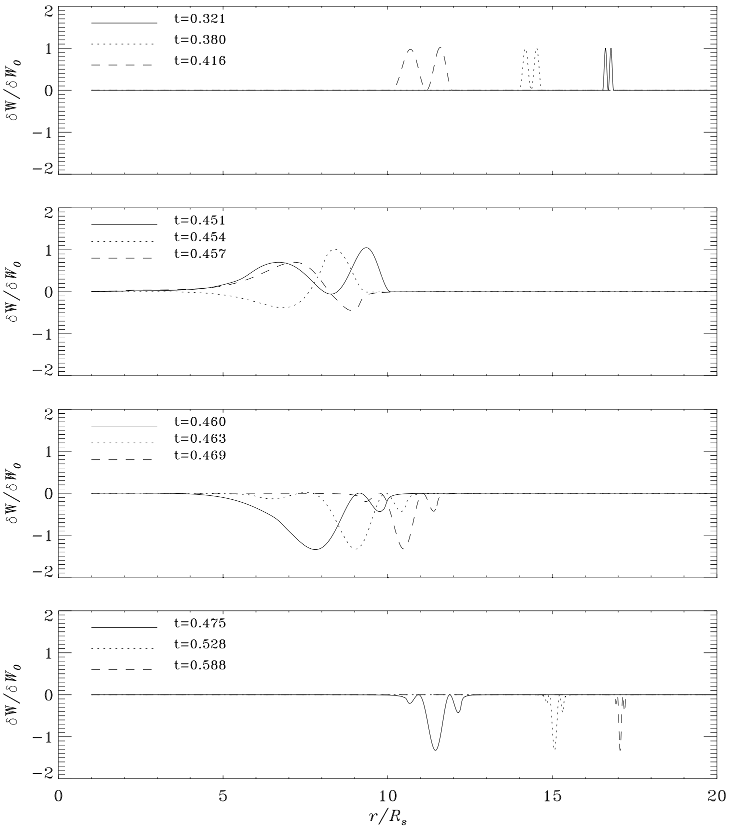

Figure 4: Wave energy density as a function of radial distance for different times.

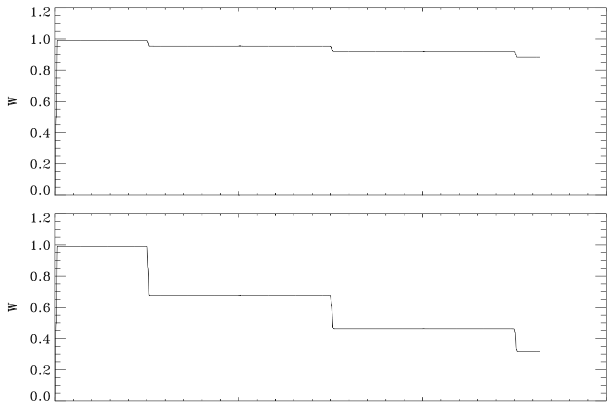

Figure 5: Relaxation of the Alfvén wave energy in the dipole magnetic tube for the meridional (top) and azimuthal (bottom) polarizations of the velocity perturbation in the Alfvén wave. The time is given in units of the double propagation time from the equator until the reflection zone.

is caused by the influence of the conducting ionosphere. The wave energy is constant during the propagation of the wave pulse until the boundary, and it decreases sharply after each reflection. The wave energy is normalized to its initial value. One can see from the figure that the decrease of the wave energy is much less pronounced for the case of meridional polarization. However, for both cases, the wave energy decrease is much less than it could be in a straight magnetic tube. The convergence of magnetic field lines substantially diminishes the loss of the wave energy.

The figures indicate clearly that the length scale of Alfvén perturbations propagating along the converging dipole magnetic field lines increases substantially. The reflection zone is determined from the condition that the length scale of the wave pulse becomes of the order of the distance to the conducting boundary. The wave pulse is reflecting mainly from a narrow spot of the dipole magnetic tube, and thus only the edge of the pulse can reach the ionosphere. Because of that, the wave energy flux to the ionosphere is rather small, and the dissipation of the wave pulse is very weak irrespective to a finite conductivity of the ionosphere.

New magnetic field data from Cassini flybys at Titan were analyzed and compared with model simulations by Backes et al. [2005]. From comparison, they suggest the incident flow plasma conditions to be not substantially different from those of Voyager 1. However the trajectory of Cassini was not favorable for detecting the Alfvén wings appearing in the numerical simulations.

4 Conclusions

Our results of Alfvén wave propagations concern mainly the effects caused by the convergence of dipole-like magnetic field lines in the magnetosphere of Saturn. Numerical simulations show that the length scale of Alfvén perturbations, propagating along the converging magnetic field lines, increases proportional to the magnetic field strength. The reflection stage starts as soon as the length scale of the wave pulse becomes of the order of the distance to the ionosphere of Saturn. The wave pulse is reflected from a very narrow part of the magnetic tube, and thus it does not reach the ionosphere of Saturn. The wave energy flux to the ionosphere is rather small, and thus the dissipation of wave perturbations is very weak. Because of that, the Alfvén wave pulses have many reflections from the ionosphere without a noticeable damping. This means that the wave damping is diminished by the effect of the magnetic field convergence, which is more pronounced for the meridional wave polarization, than for the azimuthal one. This effect is also stronger when the initial length scale of the pulse is larger.

Acknowledgements

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References

Backes, H. , F. M. Neubauer, M. K. Dougherty, N. Achilleos, N. Andre, C. S. Arrige, C. Bertucci, G. H. Jones, K.K. Khurana, C.T. Russell, A. Wennmacher, Titan's magnetic field signature during first Cassini encounter, *Science*, **308**, 992–995, 2005.

Erkaev, N. V., V. A. Shaidurov, V. S. Semenov, D. Langmayr and H. K. Biernat, Peculiarities of Alfvén wave propagation along a nonuniform magnetic flux tube, *Phys. Plasmas*, **12**, 012905(1-12), 2005.

Klimushkin, D. Y., Theory of azimuthally small scale hydromagnetic waves in the axisymmetric magnetosphere with finite plasma pressure, *Ann. Geophys.*, **16**, 303–321, 1998.

Langmayr, D., N. V. Erkaev, V. S. Semenov, W. Macher, H. K. Biernat, H. O. Rucker, Analysis of a pressure disturbance in a homogeneous magnetic field, *Adv. Space Res.*, **33**, 780—783, 2004.

Ledvina, S. A., J. G. Luhman, S. H. Brecht, T. E. Cravens, Titan's induced magnetosphere, *Adv. Space Res.*, 33, 2092–2102, 2004.

Leonovich A. S. and V. A. Mazur, An electromagnetic field, induced in the ionosphere and atmosphere and on the Earth's surface by low-frequency Alfvén oscillations of the magnetosphere: General theory, *Planet. Space Sci.*, **39**, 529-545, 1990.

Neubauer F. M., D. A. Gurnett, J. D. Scudder, and R. E. Hartle, Titan's magnetospheric interaction, In: *Saturn*, edited by T. Gehrels and M. S. Matthews, University of Arizona Press, 760-787, 1984.