TRANSIONOSPHERIC PROPAGATION PARAMETERS CALCULATED FROM EMPIRICAL ELECTRON DENSITY MODELS ADAPTED TO REALISTIC CONDITIONS

R. Leitinger*

Abstract

The "family" of "profiler" type models developed at the Institute for Geophysics, Astrophysics and Meteorology (IGAM) of the University of Graz and at the International Centre for Theoretical Physics (ICTP) at Trieste [*NeQuick*, *COSTprof* and *NeUoG–plas*, see, e.g. Hochegger et al., 2000] as well as the International Reference Ionosphere [IRI, see, e.g., Bilitza, 2001] allow to calculate all propagation parameters along chosen ray paths. As shown in the accompanying paper [Leitinger et al., 2006] the models can be updated to actual or to "realistic" conditions. They allow to calculate all propagation parameters (transionospheric propagation errors) along arbitrarily chosen ray paths.

This paper deals with transionospheric propagation errors on the basis of the refractive indices for "monochromatic" radio waves in a magneto–plasma. The refractive indices are expanded in $(1 / f)$ series (f being transmitted signal frequency). Suitable signal combinations then provide the observable propagation effects.

Because of the importance for ground reception of planetary radio waves this paper concentrates on the Faraday effect. For signal frequencies < 30 MHz it is recommended to take into account higher order correction terms. For a signal frequency of 20 MHz examples are shown of the (first order) Faraday effect and of higher order terms both for vertical and for slant ray paths.

1 Introduction: Ionospheric propagation errors

Ionospheric propagation errors are a long standing problem which has got new importance in connection with satellite based navigation systems like WAAS[1] for the US (see, e.g., El–Arini et al., 2001) or EGNOS[2] for Europe. First order errors have to be assessed in

* *Institute of Physics, Department for Geophysics, Astrophysics und Meteorology (IGAM), University of Graz*

[1] Wide Area Augmentation System (WAAS), see `http://gps.faa.gov/Programs/WAAS/waas.htm`

[2] European Geostationary Navigation Overlay Service, see `http://www.esa.int/esaNA/index.html`

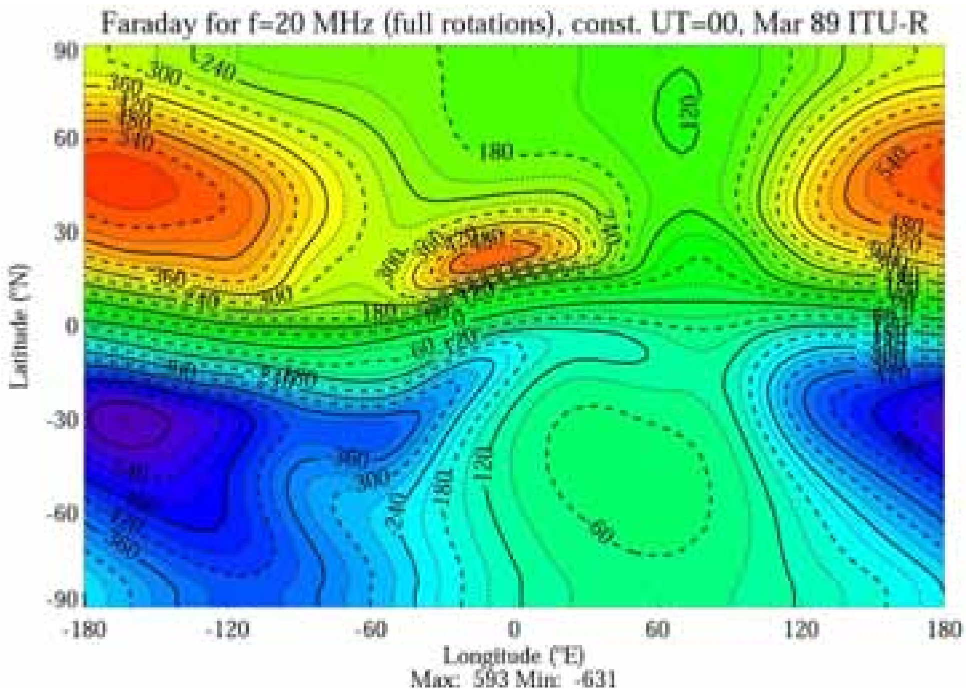

Figure 1: Global distribution of first order vertical Faraday rotation for March 1989.

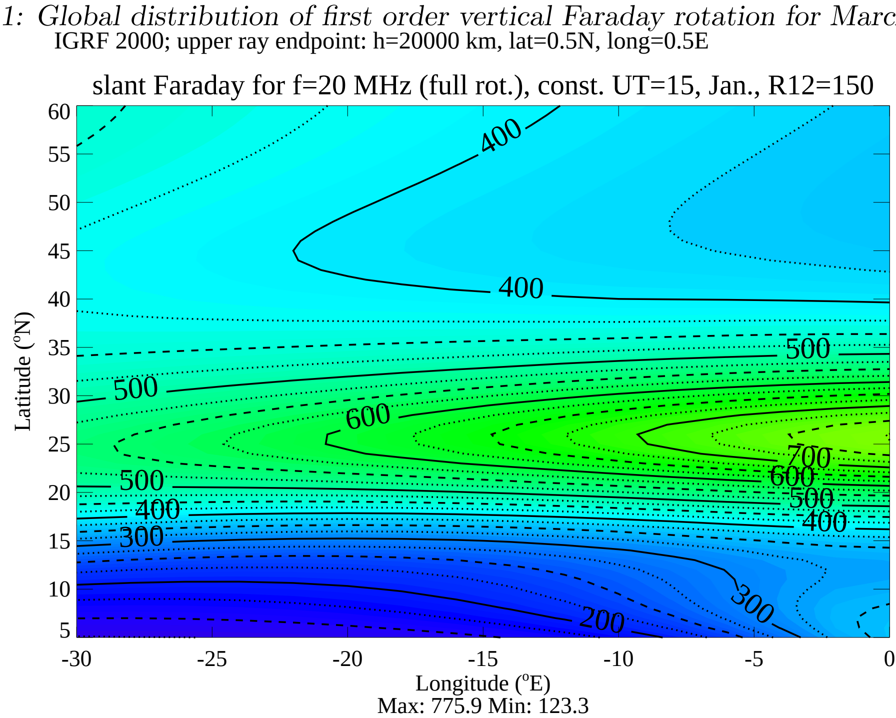

Figure 2: Regional distribution of first order slant Faraday rotation for January and high solar activity ($R_{12} = 150$).

the case of "single frequency users", higher order errors are (and will be) important in all longterm applications, like geodynamics, but should also be carefully considered in the case of high precision positioning.

Nowadays propagation error assessment uses electron density models [Leitinger et al.,

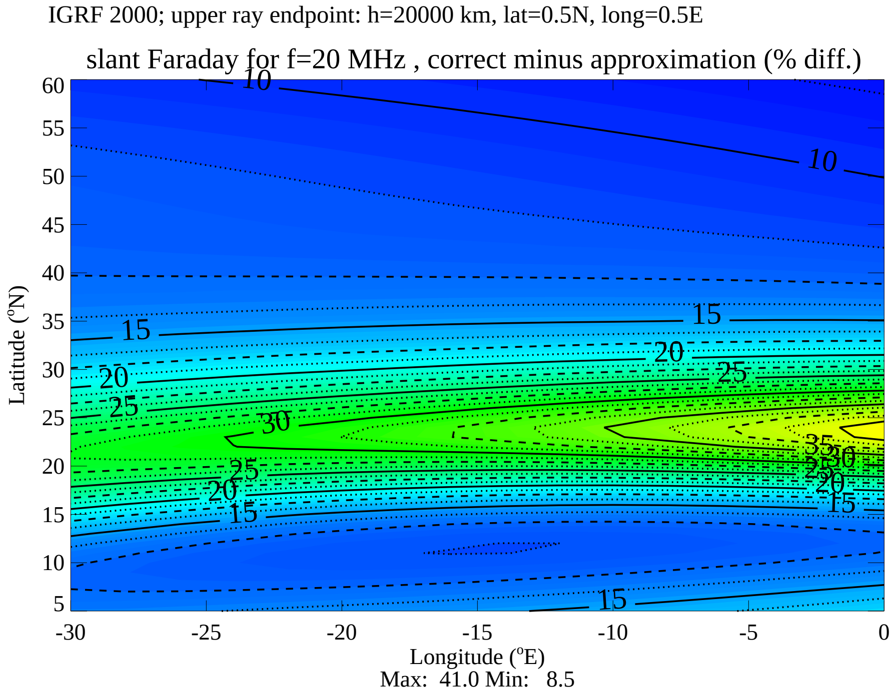

Figure 3: Percent deviation between complete formula slant Faraday rotation (integration along straight lines) minus first order slant Faraday rotation. Conditions as Fig. 2.

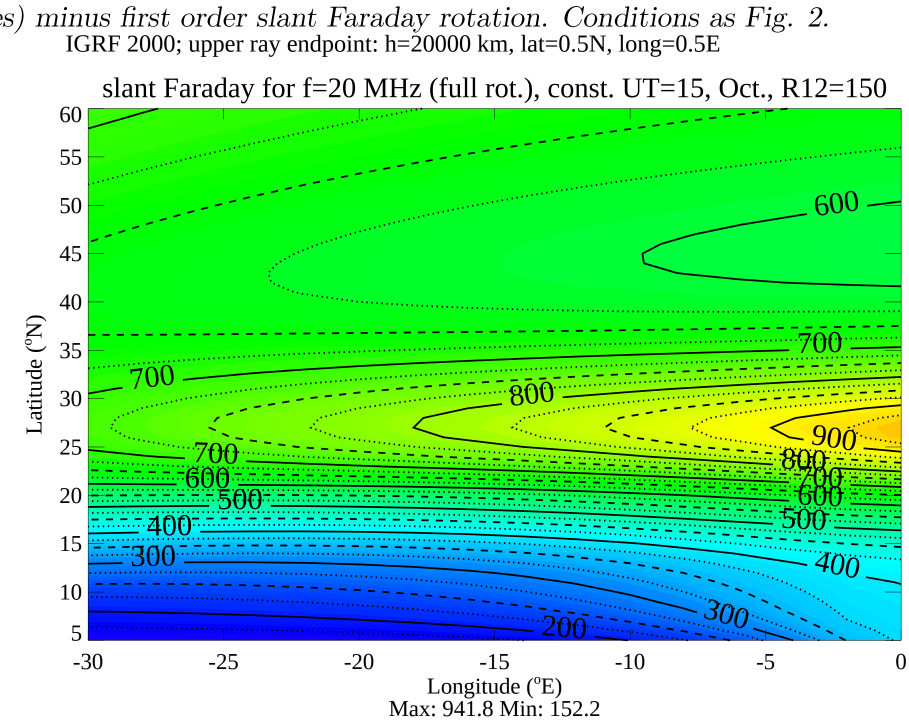

Figure 4: Regional distribution of first order slant Faraday rotation for October and high solar activity ($R_{12} = 150$).

2006] and this new development helps with longstanding problems, like precise calculation of the Faraday effect which affects planetary radio signals observed from the ground.

The starting point is the dispersion formula for a cold magneto–plasma for frequencies

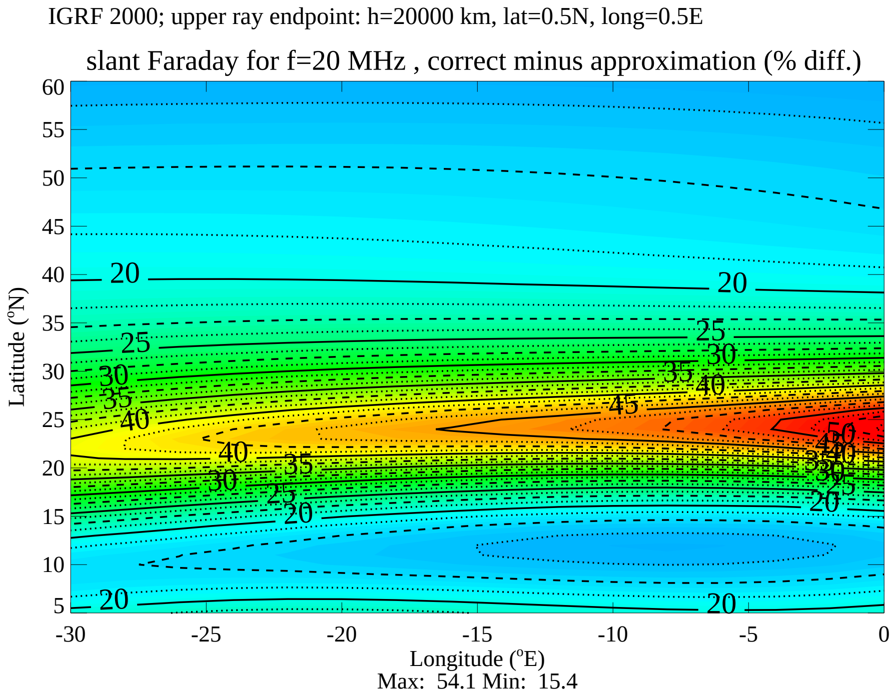

Figure 5: Percent deviation between complete formula slant Faraday rotation (integration along straight lines) minus first order slant Faraday rotation. Conditions as Fig. 4.

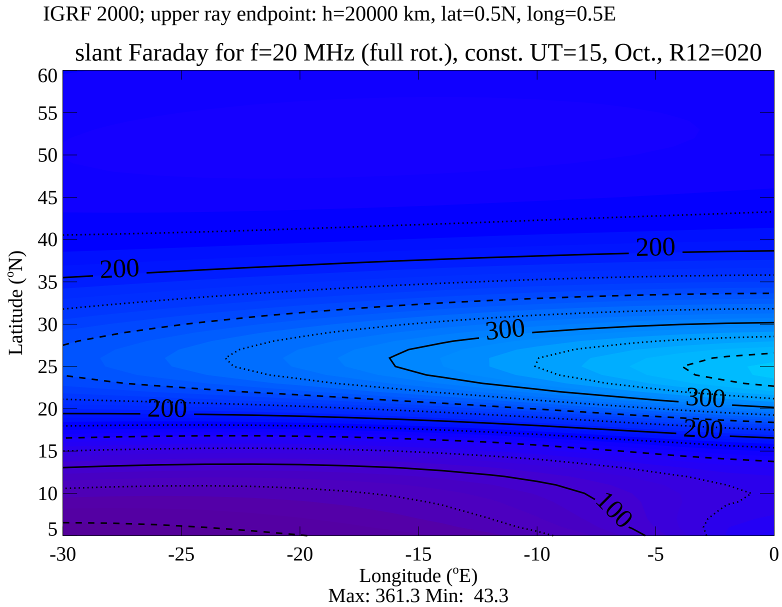

Figure 6: Regional distribution of first order slant Faraday rotation for October and low solar activity ($R_{12} = 20$).

substantially higher than the gyrofrequencies of the ions (e.g., [Leitinger, 1992]):

$$n^2 = 1 - \frac{\tilde{X}(1-\tilde{X})}{1 - \tilde{X} - \tilde{Y}_T^2/2 \pm \sqrt{\tilde{Y}_T^4/4 + \tilde{Y}_L^2(1-\tilde{X})^2}}$$

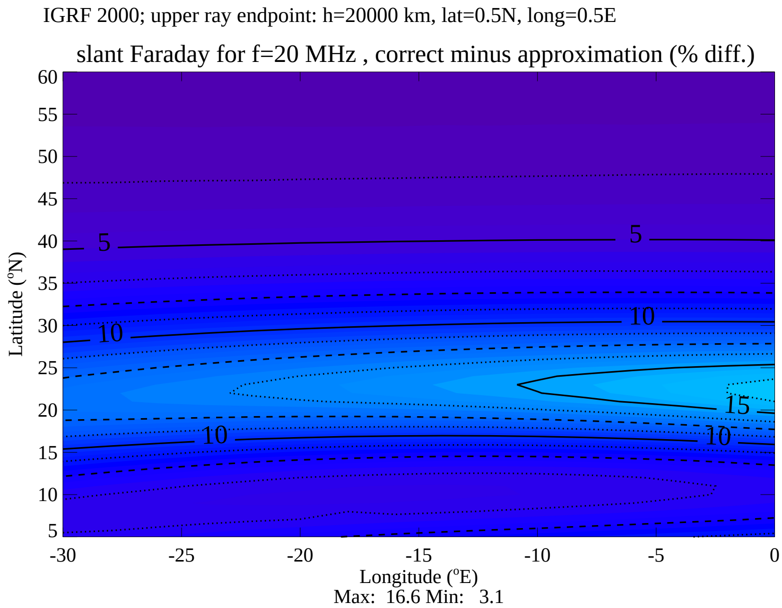

Figure 7: Percent deviation between complete formula slant Faraday rotation (integration along straight lines) minus first order slant Faraday rotation. Conditions as Fig. 6.

with $\tilde{Y}_L = \tilde{Y}|\cos\Theta|$ (longitudinal component), $\tilde{Y}_T = \tilde{Y}\sin\Theta$ (transversal component),

$$\tilde{X} = \frac{X}{1+jZ}, \qquad \tilde{Y} = \frac{Y}{1+jZ}, \qquad j = \sqrt{-1}\,.$$

Conventional acronyms: $X = f_p^2/f^2$, $Y = f_g/f$, $Z = \nu/(2\pi f)$, $f_p^2 = (e^2N)/(4\pi^2 m\epsilon_o) = A\,N$, $f_g = e/(2\pi\, mB)$;
f_p: electron plasma frequency, f_g: electron gyrofrequency, ν: effective collision frequency for electrons, f: transmitted frequency, N: electron density, e: electron charge, m: electron mass, ϵ_o: permittivity of free space, B: geomagnetic induction. With S.I.–units the value of A is 80.6.

Refractive indices are defined for the "principal polarizations" only. In the geomagnetic Northern hemisphere the $+$ sign in the nominator corresponds to the lefthand elliptical component, the $-$ sign to the righthand elliptical component.

We assume now high frequencies ($f \gg f_p$, $f \gg f_g$) and can neglect attenuation (omit Z — omit the tilde $\tilde{}$). In the quasi–longitudinal approximation the dispersion formula is then

$$n_{1,2}^2 = 1 - \frac{X}{1 \pm Y_L - Y_T^2/2}$$

for nearly circular principal polarizations ($+$ sign: lefthand, $-$ sign: righthand in the geom. Northern hemisphere).

For the Faraday effect we need the difference of refractive indices $n_1 - n_2$. By means of $n_1^2 - n_2^2$ and $n_1 + n_2 \doteq 2 - X$ it can easily be shown that the "first order Faraday effect" is proportional to $\int_{\mathrm{R}}^{\mathrm{S}} X\,Y_L\,\mathrm{d}s_o$, S: (satellite) transmitter, R: (ground) receiver, $\mathrm{d}s_o$: path element along the straight line from S to R.

For comparatively low frequencies (for $f < 30$ MHz) we should account for higher order influences, too.

If we replace the ray paths by the wave normals (neglecting anisotropy, replacing the "real" ray paths by the common "isotropic" ray path) we can assume that the Faraday effect is proportional to $\int_{\mathrm{R}}^{\mathrm{S}} (n_1 - n_2)\mathrm{d}s$, $\mathrm{d}s$ being the "isotropic" path element and we can use the error considerations of [Leitinger, 1992]

$$\int_{\mathrm{R}}^{\mathrm{S}} (n_1 - n_2)\mathrm{d}s = \int_{\mathrm{R}}^{\mathrm{S}} (n_1 - n_2)\mathrm{d}s_o + \left[\int_{\mathrm{R}}^{\mathrm{S}} (n_1 - n_2)\mathrm{d}s - \int_{\mathrm{R}}^{\mathrm{S}} (n_1 - n_2)\mathrm{d}s_o\right]$$

The term in [ ] is the "ray bending" error not considered here.

$\int_{\mathrm{R}}^{\mathrm{S}} (n_1 - n_2)\mathrm{d}s_o$ can easily be calculated by means of numerical integration on the basis of an empirical electron density model updated to realistic conditions.

2 Examples: Faraday effect for $f = 20$ MHz

The first example (Figure 1) shows the global distribution of "vertical" first order Faraday rotation for $f = 20$ MHz for the "nominal ionosphere" under high solar activity conditions (March, 1989). *NeQuick* was used with the "CCIR" maps for foF2 and M(3000)F2 and the geomagnetic field model IGRF1990. Vertical rays have been assumed (radio wave incidence from the local zenith).

The other examples (Figures 2 through 7) show regional distributions of first order Faraday rotation (Figures 2, 4 and 6) for slant rays, the upper ray endpoint being at a height of 20000 km and at geographic latitude 0.5°N and longitude 0.5°E. Figures 3, 5 and 7 give the percent deviations between the slant Faraday rotation calculated with the complete dispersion formula minus first order Faraday rotation.

For $f = 20$ MHz and high solar activity conditions (Figures 2 through 5) the percent differences between "full formula" and "first order" Faraday rotation are substantial and should not be neglected in the vicinity of the equatorial anomaly. In our October example the percent differences exceed 50% (Figure 5). Still higher differences cannot be excluded under disturbed ionospheric conditions (storm enhancements of ionospheric electron content).

In the vicinity of the equatorial anomaly the percent deviations can be substantial even under low solar activity conditions (Figure 7).

3 Conclusions

Realistic electron density models are a powerful tool for the prediction of radio wave propagation effects, for assessment studies but also for the proper interpretation of observations.

As shown here even monthly median models can give important information about propagation effects, e.g., the solar activity and geometric limits for the safe application of the first order Faraday formulae.

Since all members of the "family" of 3D and time dependent electron density models developed at Graz and at Trieste can be updated to actual (observed) and to realistic conditions much more can be done. Consequently, we plan to repeat Faraday effect calculations for actual conditions, e.g., for geomagnetic storm situations.

In the frame of the COST[3] Action 296 "Mitigation of Ionospheric Effects on Radio Systems (MIERS)" it is also planned to assess long term effects like systematic ionospheric influences on geodetic applications of radio waves transmitted from Global Navigation Satellites.

References

Bilitza, D., International Reference Ionosphere 2000, *Radio Sci.*, **36**, 261–275, 2001.

El–Arini,M.B., W. Poor, R. Lejeune, R. Conker, J. Fernow, and K. Marini, An introduction to Wide Area Augmentation System and its predicted performance. *Radio Sci.*, **36**, 1233–1240, 2001.

Hochegger, G., B. Nava, S. Radicella and R. Leitinger, A family of ionospheric models for different uses. *Phys. Chem. Earth (C)*, **25**, 307–310, 2000.

Leitinger, R.: Propagation effects influencing the observations of planetary radio emissions. in *Planetary Radio Emissions III*, H. O. Rucker, S. J. Bauer, and M. L. Kaiser (eds.), Austrian Academy of Sciences Press, Vienna, 171–181, 1992.

Leitinger, R., E. Feichter, M. Rieger and C. Martinecz, Adapting empirical electron density models to disturbed conditions, in *Planetary Radio Emissions VI*, H. O. Rucker, W. S. Kurth, and G. Mann (eds.), Austrian Academy of Sciences Press, Vienna, 2006, *this issue*.

[3] COST: European Cooperation in the Field of Scientific and Technical Research