MATHEMATICAL MODEL OF MAGNETIC FIELD PERTURBATIONS BY CURRENTS IN THE EARTH'S MAGNETOSPHERE

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Abstract

A mathematical model concerning magnetic field perturbations is elaborated. It is based on the numerical solution of a 3–D problem of steady state magnetostatics inside an arbitrary domain. The normal component of the magnetic field at the magnetopause and on the Earth's surface and the space distribution of the electric current density ought to be given. Both, scalar and vector potentials, are used to prove the variational principle and to create a finite element method. A multigrid method is used for obtaining the effective solution of the system of linear algebraic equations for the grid values of the potentials.

The designed model is applied to calculate magnetic field perturbations due to current systems in the Earth's magnetosphere. It is shown that the magnetopause shifts closer to the Earth as a result of field–aligned currents in the auroral zone and this distance is estimated. The model is also used to trace magnetic field lines, which is necessary for mapping of the magnetospheric regions of electric field generation onto the ionosphere where the electric energy is dissipated.

Thus, the presented model contributes essentially to the understanding of the chain of magnetic field variations in the solar wind, perturbation of the magnetospheric field and current systems, and thus, the influence on the magnetospheric radio emission process.

1 Introduction

One of the main properties of the near space is the magnetic field distribution. Any physical model of electric field generation, plasma motion and waves propagation and generation needs space distribution of the magnetic field. A steady–state magnetic field may be calculated if the space distribution of electric currents and boundaries are known.

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The purpose of the paper is to present the designed mathematical model and to demonstrate how it works in the analysis of magnetic field perturbations, which are produced by the main field-aligned current systems [Iijima and Potemra, 1978]. The main electric nets in the Earth's magneto-sphere includes generators inside the magnetosphere and electric energy dissipation in the ionosphere [Kamide and Matsushita, 1979]. These domains are connected by field-aligned currents. Since these currents are rather week, the undisturbed model of the magnetospheric magnetic field lines can be used. Then, the magnetic field perturbation due to field-aligned currents can be calculated. Some iterations may be done for precision.

2 Magnetic field calculation

A steady state magnetic field B satisfies the equations of magneto-statics,

$$\begin{aligned} \text{curl}\ \mathbf{B} &= \mu_0 \mathbf{j} \\ \text{div}\ \mathbf{B} &= 0 \end{aligned} \tag{1}$$

with a given electric current density $\mathbf{j}$ inside the magnetosphere.

If there is no magnetic reconnection, then the magnetopause M is considered as tangential discontinuity, hence, the normal component of the field is equal to zero. The magnetic field may have a nonzero component, B_n^0, that is the result of a model of reconnection,

$$B_n|_M = B_n^0, \tag{2}$$

where index n marks the normal component of the vector. The radial component of the field is equal to the undisturbed value at the Earth's surface,

$$B_r|_{r=1} = B_r^0. \tag{3}$$

If we take into account only the geomagnetic dipole and if it is parallel to the $z-$axis, then $B_r^0(\theta, \varphi) = B_0 \cos(\theta)$, where $B_0 = 0.6$ gauss. We use the GSM coordinates x, y, z with the Earth's radius R_E as the length unit and corresponding spherical coordinates r, θ, φ.

There also must be put some condition in the far tail. If we are interested in the head part of the magnetosphere, this is not so important, because its influence decreases with the scale of about the diameter of the tail. We suppose, that the magnetic field becomes parallel to the $x-$axis,

$$B_{y,z}|_{x=-x_\infty} = 0. \tag{4}$$

For this purpose we must preliminary assume that the right-hand side function B_n^0 in (2) becomes zero at that distance as well as the current density j_x in (1).

If we need a model with a geomagnetic dipole inside an empty magnetosphere, a magnetic potential F may be introduced, so that

$$\mathbf{B} = -\text{grad} F. \tag{5}$$

Then the problem (1-4) may be reduced to the following problem for the Poisson equation, since $-\text{div grad}F = -\Delta F$

$$-\Delta F = 0, \quad -\frac{\partial F}{\partial n}\bigg|_M = B_n^0, \quad -\frac{\partial F}{\partial r}\bigg|_{r=1} = B_r^0, \quad F|_{x=-x_\infty} = 0. \tag{6}$$

It is usual to reduce the boundary value problem (1-4) to the problem for a vector potential $\mathbf{A}$, but we use a pair of potentials, a scalar F and a vector $\mathbf{A}$ simultaneously,

$$\mathbf{B} = -\text{grad } F + \text{curl } \mathbf{A}, \tag{7}$$

with the following conditions for $\mathbf{A}$,

$$\text{div } \mathbf{A} = 0, \quad \mathbf{A}_\tau|_M = 0, \quad \mathbf{A}_{\theta,\varphi}|_{r=1} = 0, \quad A_x|_{x=-x_\infty} = 0, \tag{8}$$

where τ marks tangential components of a vector. Then the problems for F and $\mathbf{A}$ are separated. The same problem given by (6) for F and three Poisson equations for Cartesian components of $\mathbf{A}$ are

$$-\Delta A_x = \mu_0 j_x, \quad -\Delta A_y = \mu_0 j_y, \quad -\Delta A_z = \mu_0 j_z \tag{9}$$

with the conditions (8) which involve all components of $\mathbf{A}$.

If we use only a vector potential, its tangential components can not be zero, because their curl equals to the given normal component of the magnetic field, in contrast with the second condition (8). It permits us to use a minimization of the energy functional as a way to solve the problem.

In accordance to the Dirichlet principle for the Poisson equation, one can obtain the solution F for the problem (6) when one reaches minimum of the energy functional,

$$w(F) = \frac{1}{2}\int (\text{grad}F)^2 \, dxdydz + \int_M FB_n^0 \, dM + \int_{r=1} FB_r^0 \sin(\theta) \, d\theta d\varphi \tag{10}$$

at the set of functions which satisfy only the last condition of (6). We can get a solution $\mathbf{A}$ for the problem (9, 8), when we reach the minimum of the energy functional,

$$W(\mathbf{A}) = -\int ((\text{curl } \mathbf{A})^2 + (\text{div } \mathbf{A})^2 - 2\mu_0 \mathbf{A}\mathbf{j}) \, dxdydz \tag{11}$$

at a set of functions which satisfy only the last three conditions from (8). This principle is proved in [Denisenko V.V., 1997] when the tangential components are equal to zero at the whole boundary. The problem (9, 8) becomes one of this sort if we add a symmetrical domain besides of the plane $x = -x_\infty$ and exclude this part of the boundary. The solution in that doubled domain satisfies the last condition (8) because of symmetry.

Potential presentation of the magnetic field can be also used if currents are given at some surface inside the domain. The potential F has a jump at such a surface:

$$F(x, y, +0) - F(x, y, -0) = 2F^0(x, y), \tag{12}$$

where $2F^0(x, y)/\mu_0$ is the current function of the currents in the neutral layer. We use the model by Kitaev [1996] of these currents. The condition (12) must be added to the conditions (6) as well as to the conditions for the energy functional (10) minimization. For the consistency of the condition (12) with the last condition (6), currents in the far tail must have $y-$ direction that is already supposed above. Then, we can put $F^0(-x_\infty, y) = 0$.

To obtain numerical solutions for the problems (6) or (9, 8) we use a version of the finite element method. It is based on the minimization of the energy functional (10) or (11) at piece-wise linear functions. We divide the domain of interest into a few subdomains each of which is similar to a curvilinear parallelepiped and design a regular nonuniform grid in each of them. Every grid cell is divided into tetrahedra to construct piece-wise linear functions. The multigrid method by Fedorenko is used to solve our systems of linear algebraic equations which arrive for the nodal values of approximating functions. The details of our finite element method are presented in [Denisenko, 2004]. The first results of the model application for the calculation of the magnetospheric magnetic field perturbation caused by cusp currents are presented in [Denisenko, et al., 2004].

3 Field-aligned currents in the auroral zone

The described mathematical model is used here to analyze typical magnetic field perturbation due to field-aligned currents in the auroral zone. The most of them are the currents in zones 1 and 2 [Iijima and Potemra, 1978], [Kamide and Matsushita, 1979].

We define a typical distribution of field-aligned currents in zone 1 [Iijima and Potemra, 1978] as they are presented by the bold lines in Figure 1. Nonzero currents are in a strip that is between $\theta = 12^o$ and $\theta = 16^o$ at midday and between $\theta = 20^o$ and $\theta = 24^o$ at midnight. The total current equals to ± 1 MA. Nonzero currents of zone 2 are in a strip that is between $\theta = 17^o$ and $\theta = 20^o$ at midday and between $\theta = 25^o$ and $\theta = 28^o$ at midnight. The total current equals to ± 0.5 MA.

Figure 1 also presents field aligned currents after their mapping to the equatorial plane. These currents are closed through the plane $z = 0$. Currents of the zone 1 are continued to the nearest point at the magnetopause along arcs with the center at $x = 10.4$, $y = z = 0$. Then they are automatically closed along magnetopause because of the first condition (6). Currents of the zone 2 are closed to each other along arcs with the center at $x = y = z = 0$.

Since $j_r = 0$ in the main part of the domain, it's impossible to plot the line $j_r = 0$. So we start with two lines corresponding to $|j_r|$ equal to a half of the contour interval and plot the remaining lines with given contour intervals. The same manner is used for j_z presentation in Figure 1.

The solution for the problem (9, 8) gives our model of magnetic field perturbations. We do this separately for the currents of the zone 1 and zone 2. The results are presented in Figures 2,3. The distributions of B_z perturbation are shown. Other components are equal to zero at this plane because of the symmetry.

Figures 2,3 demonstrate that the main perturbations are concentrated near the currents in the $z = 0$ plane, which close the field-aligned current system.

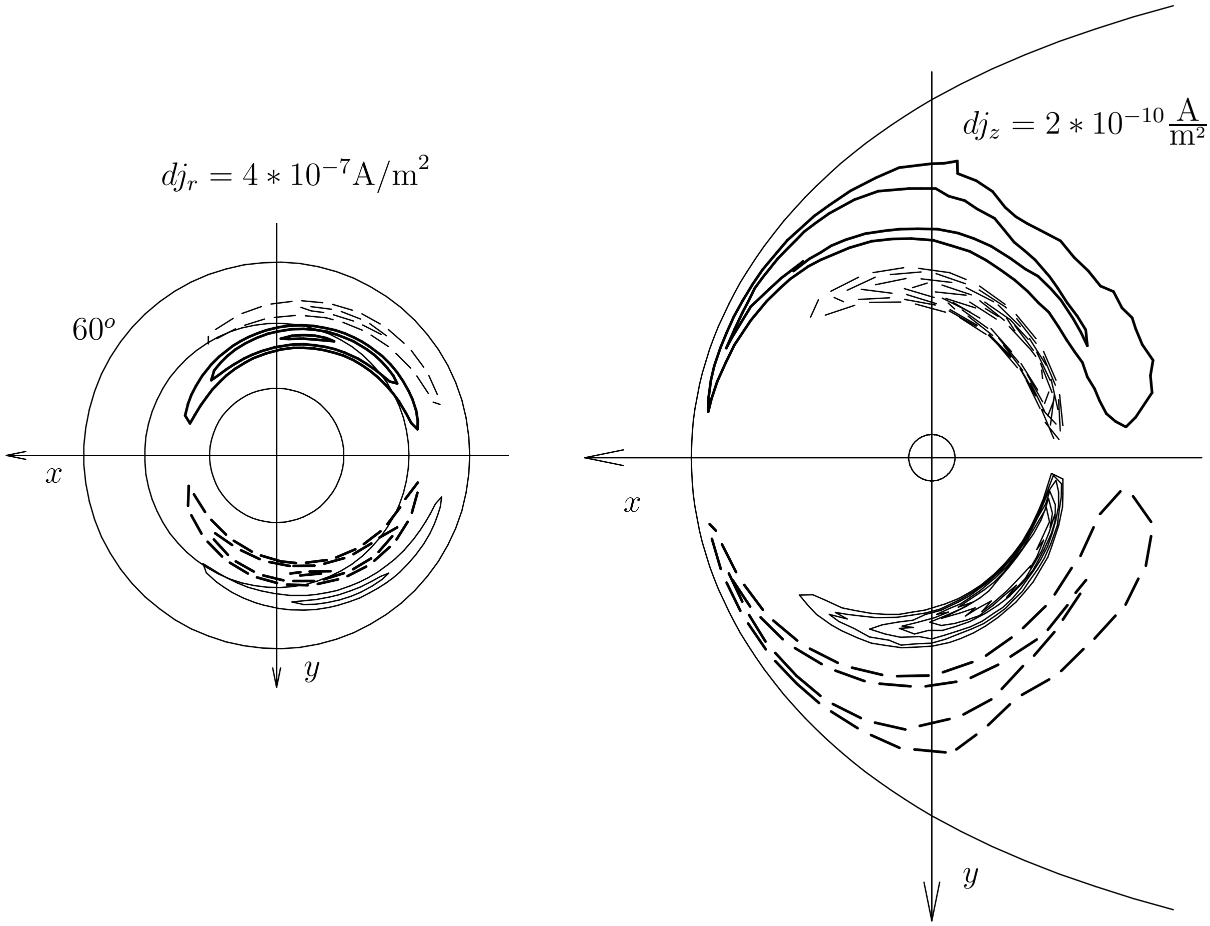

Figure 1: Distributions of j_r in the ionosphere (left panel), and j_z at the plane $z = 0$ (right panel), for zone 1 and 2 field-aligned currents. Bold lines present zone 1 currents. Fine lines present zone 2 currents. Negative values are presented as dashed lines. The head part of the magnetopause and the Earth's surface are plotted at the right panel to show the domain.

The currents of zone 1 have more influence on the magnetic field near the magnetopause because their closing currents are there, and the currents of zone 2 produce much lager perturbation in the night sector because they have some loop around this area.

The magnetic field at the subsolar point must satisfy the condition of pressure balance. Therefore it is known for given parameters of the solar wind. This means that the magnetopause must be shifted closer to the Earth as a result of the negative B_z perturbation. Such a shift, is referred as the erosion of the magnetopause.

If we suppose geometric similarity, the erosion may be estimated to be 1000 km for B_z perturbation of about -3 nT, so that the currents of zone 1 form, as it is seen in Figure 2. Magnetic field perturbations are maximal at the flanks of the magnetopause, δB_z varies between -5 nT and +10 nT. These perturbations vary the shape of the magnetopause. The magnetopause shifts closer to Earth in front of the point $x = 2$ and further from the Earth behind this point. Figure 3 shows that the currents of zone 2 are not effective as far as erosion of the magnetopause is concerned.

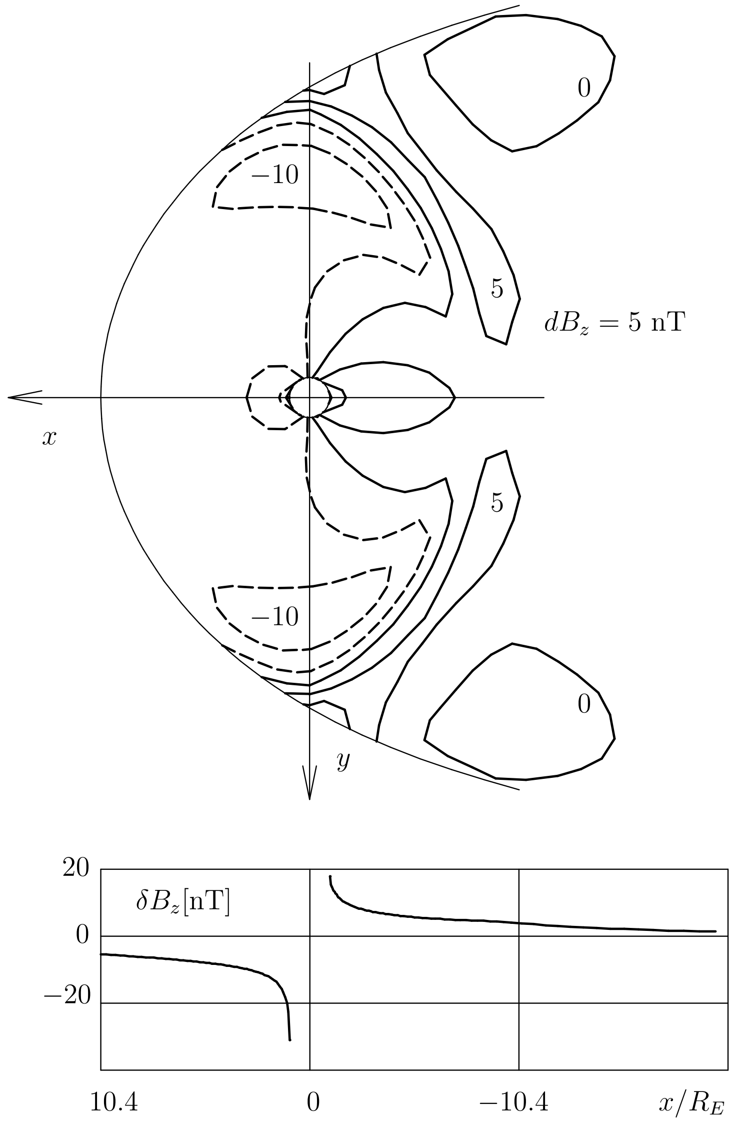

Figure 2: Distributions of δB_z at the plane $z = 0$ (top panel) and along the x axis (bottom panel) for the zone 1 currents. The head part of the magnetopause and the Earth's surface are plotted to show the domain. Negative values are presented with dashed lines.

4 Conclusions

The presented mathematical model of magnetic field perturbations is used to estimate magnetic field perturbations which are produced by the main field-aligned current systems. It is shown that this currents with their closure can produce perturbations up to 40 nT in the equatorial plane.

Only the currents of zone 1 perturb the magnetic field near the magnetopause. Mainly due to their closing currents which in this respect are similar to other currents across the magnetospheric tail. They define erosion of the magnetopause at the subsolar point of about 1000 km and a few times larger erosion at the flanks.

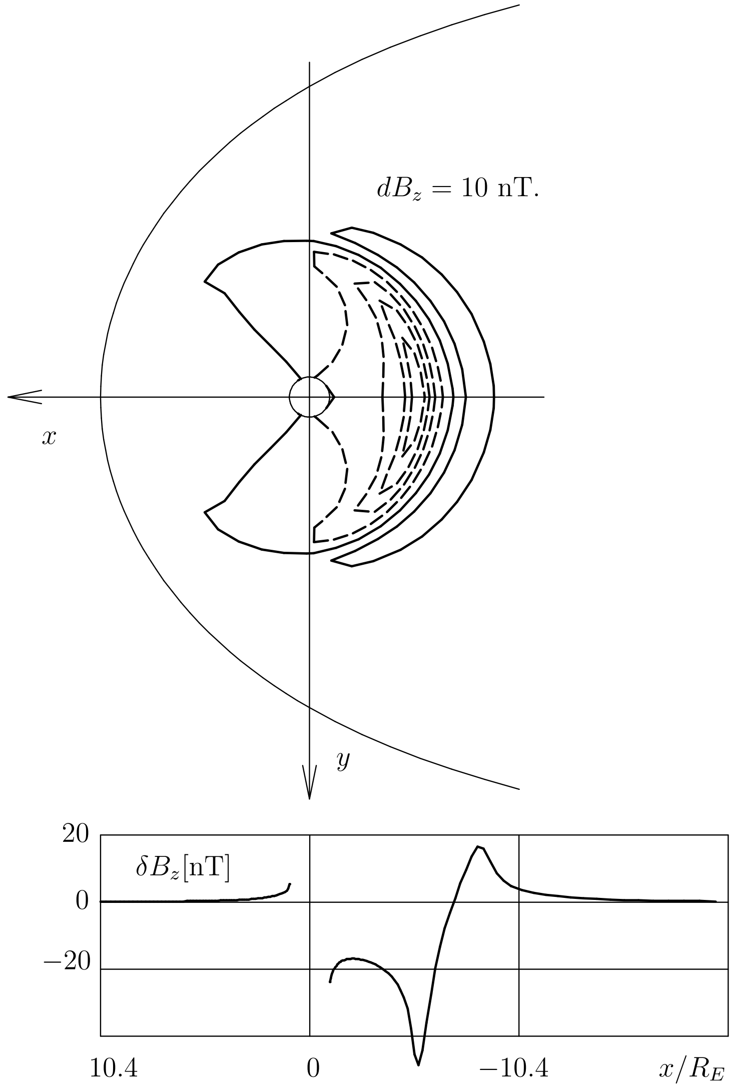

Figure 3: Distributions of δB_z at the plane $z = 0$ (top panel) and along the x axis (bottom panel) for the zone 2 currents.

Another point of view appears if we take into account that zone 1 field-aligned currents appear as the result of the brake of the currents in the neutral layer. Then we are to regard as the perturbation the difference between this current system and currents which existed before this break. Such a system is similar to the system of zone 2 currents and so it perturbs magnetic field in some part of the magnetosphere in the night sector only.

The erosion at the subsolar point due to the currents of zone 1 is twice more than one due to typical cusp currents, as it was estimated in [Denisenko, et al., 2004]. These estimations show that real erosion of the magnetopause ought be explained mainly by other current systems. Currents across the magnetospheric tail in the neutral layer seem to be dominant.

The presented mathematical model of magnetic field perturbations can be used for such a research as described. It as well permits to include any current system to the model of the magnetic field for magnetospheres of Earth and other planets.

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References

Denisenko V.V., Energy methods application for vector fields calculation. *Journal of Computational Technologies*, **9**, 42-56, 2004. (in Russian).

Denissenko V.V., The energy method for three-dimensional elliptic equations with asymmetric tensor coefficients, *Siberian Mathematical Journal*, **38**, 1099–1111, 1997.

Denisenko V.V., N.V. Erkaev, V.S. Semenov, S.S. Zamay, and H.K. Biernat, Calculation of the Magnetospheric Magnetic Field Perturbation Caused by the Cusp Currents. *Solar-Terrestrial physics. V. 8. Proceedings of the International Conference on the Physics of Solar-Terrestrial Relations. Irkutsk, 2004*, 126–128. 2005.

Iijima T., and T.A. Potemra, Large-scale characteristics of field aligned currents associated with substorms, *J. Geophys. Res.*, **83**, 55, 1978.

Kamide Y., and S.J. Matsushita, Simulation studies of ionosphere electric fields and currents in relation to field-aligned current, *J. Geophys. Res.*, **84**, 4083, 1979.

Kitaev A.V., A model of closed magnetic field of the Earth magnetosphere, *Geomagnetism and Aeronomy*, **36**, 154-15, 1996.