INFLUENCE OF THE ION FLOW DIRECTION ON THE MODIFIED TWO STREAM INSTABILITY

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Abstract

This study focuses on the modified two stream instability (MTSI) in a two component plasma within the traditionally applied approximation of magnetized electrons and unmagnetized ions. A particular coordinate system is adopted in which electrons are at rest and ions move at an arbitrary angle α across the magnetic field. Special emphasis is given on the influence of this angle on the growth rate of the MTSI. In particular, we investigate the location and the amount of the maximum growth rate as a function of α for fixed ambient plasma parameters. For this, the full electromagnetic dispersion relation is solved numerically for the whole **k**–space in the limit of small temperatures.

1 Introduction

There are many occasions in the terrestrial magnetosphere, where ion and electron contributions have a relative velocity to each other. These occasions include the terrestrial bow shock [Matsukiyo and Scholer, 2003], the upward current regions of the auroral zone [Dombeck et al., 2001], and the current sheet in the Earth's magnetotail [Yoon and Lui, 2004]. Among the various instabilities which can be driven by such a current flowing in a plasma, this paper is devoted to the so–called modified two stream instability (MTSI). Here, the term modified refers to the case of nearly perpendicular magnetic field and direction of the relative velocity of the different particle species. This instability was first analyzed by McBride and Ott [1972] and since it's features are similar to the traditional two–stream instability a similar nomenclature has been chosen.

The stability of a plasma with respect to the MTSI is determined by various parameters including the amount and direction of the relative velocity, the ratio of the temperatures of the different plasma populations, the temperature anisotropy of the respective particle

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species, as well as the plasma beta, i.e., the ratio of magnetic to thermal pressures in the ambient plasma. Qualitatively, this instability has frequencies near the lower hybrid frequency, relatively long wavelengths, and propagate within an angle of several degrees from the relative velocity.

Considerable theoretical effort has been given to analyze the particular features of the MTSI. A complete treatise of the electrostatic approximation in the linear regime can be found in [Mikhailovskii, 1974], where all main features appearing due to a relative motion between electrons and ions are given in a very comprehensive way. The first full electromagnetic approach is given by [Lemons and Gary, 1977], where the relevant dispersion relation has been derived and solved numerically. The authors show that in general the electromagnetic effects reduce the growth rate of the MTSI. Subsequent work is performed in [Gary et al., 1987], where instabilities near the lower hybrid frequency are studied. In [Gary, 1993] a whole catalogue of kinetic instabilities is given for a wide range of plasma parameters.

The aim of the present paper is to investigate a rather theoretical aspect of the MTSI numerically, i.e, the influence of the direction of the relative velocity between electrons and ions, measured in terms of an angle α. As a test for the applied numerical procedure we initially compare our results with the analysis from [Matsukiyo and Scholer, 2003], where the special case of equally directed wave vector and bulk velocity is considered. After that we focus our interest on the behavior of the maximum growth rate as a function of the angle α in a warm two component plasma.

2 Basic equations

We adopt a cartesian coordinate system where the electrons are assumed to be at rest and the ions have a relative motion to them. The background magnetic field is taken to be along the z–axis and the bulk flow direction of the ions as well as the wave vector $\mathbf{k}$ are assumed to lie in the xz–plane inclined towards the zeroth order magnetic field under the angle α and θ, respectively. Since we consider waves with much higher frequencies than the ion gyrofrequency the ions are assumed to be unmagnetized.

After the traditional approach of linearizing the Vlasov equation, the following dispersion relation is obtained [Stix, 1992]

$$\begin{pmatrix} \epsilon_{xx} - N^2\cos^2\theta & \epsilon_{xy} & \epsilon_{xz} - N^2\cos\theta\sin\theta \\ \epsilon_{yx} & \epsilon_{yy} - N^2 & \epsilon_{yz} \\ \epsilon_{zx} - N^2\cos\theta\sin\theta & \epsilon_{zy} & \epsilon_{zz} - N^2\sin^2\theta \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0, \tag{1}$$

where

$$\epsilon = \mathbf{I} + \sum_s \chi_s, \quad N^2 = \frac{c^2k^2}{\omega^2}. \tag{2}$$

Quantities c and ω refer to speed of light and frequency of the wave, and subscript s

denotes the particle species. The electron susceptibility reads as [Stix, 1992]

$$\chi_{e,xx} = C_{e1}(1-\lambda_e)\left\{Z(\eta_{e,+1})+Z(\eta_{e,-1})\right\}, \quad (3)$$
$$\chi_{e,yy} = C_{e1}\left\{4\lambda_e Z(\eta_{e,0})+(1-3\lambda_e)(Z(\eta_{e,+1})+Z(\eta_{e,-1}))\right\}, \quad (4)$$
$$\chi_{e,zz} = C_{e2}\left\{2\omega(1-\lambda_e)(1+\eta_{e,0}Z(\eta_{e,0}))+\lambda_e((\omega+\Omega_e)(1+\eta_{e,+1}Z(\eta_{e,+1}))) \right. \left. (\omega-\Omega_e)(1+\eta_{e,-1}Z(\eta_{e,-1})))\right\}, \quad (5)$$
$$\chi_{e,xy} = iC_{e1}(1-2\lambda_e)\left\{Z(\eta_{e,+1})-Z(\eta_{e,-1})\right\} = -\chi_{e,yx}, \quad (6)$$
$$\chi_{e,xz} = C_{e3}(1-\lambda_e)\left\{\eta_{e,+1}Z(\eta_{e,+1})-\eta_{e,-1}Z(\eta_{e,-1})\right\} = \chi_{e,zx}, \quad (7)$$
$$\chi_{e,yz} = iC_{e3}\left\{(2-3\lambda_e)(1+\eta_{0,e}Z(\eta_{e,0})) \right. \left. -(1-2\lambda_e)(2+\eta_{e,+1}Z(\eta_{e,+1})+\eta_{e,-1}Z(\eta_{e,-1}))\right\} = -\chi_{e,zy}, \quad (8)$$

where

$$C_{e1} = \frac{\omega_{pe}^2}{2\omega k_z v_{te}}, \quad C_{e2} = \frac{\omega_{pe}^2}{2\omega k_z^2 v_{te}^2}, \quad C_{e3} = \frac{\omega_{pe}^2 k_x}{2\omega \Omega_e k_z}, \quad \lambda_e = \frac{k_x^2 v_{te}^2}{2\Omega_e^2}, \quad \eta_{e,n} = \frac{\omega + n\Omega_e}{k_z v_{te}}. \quad (9)$$

Here, subscript e refers to electron quantities, ω_p and Ω the plasma and gyrofrequency, and v_t the thermal velocity. The function Z refers to the so–called plasma dispersion function, which in the case of Maxwellian distributed background particles is of the following form [Mikhailovskii, 1974]

$$Z[x] = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{1}{t-x} e^{-t^2} dt. \quad (10)$$

We note that the above equations are valid as long as the relation $\lambda_e \ll 1$ is true.

The ions susceptibility is of the following form

$$\chi_{11} = \left(\frac{\omega_{pi}}{\omega}\right)^2 \eta_i Z[\eta_i] \cos^2\theta + 2\left(\frac{\omega_{pi}^2}{\omega^2}\right)^2 (1+\eta_i Z[\eta_i]) \left(\eta_i \sin\theta + \frac{u_x}{v_{ti}}\right)^2, \quad (11)$$
$$\chi_{22} = \left(\frac{\omega_{pi}}{\omega}\right)^2 \eta_i Z[\eta_i], \quad (12)$$
$$\chi_{33} = \left(\frac{\omega_{pi}}{\omega}\right)^2 \eta_i Z[\eta_i] \sin^2\theta + 2\left(\frac{\omega_{pi}^2}{\omega^2}\right)^2 (1+\eta_i Z[\eta_i]) \left(\eta_i \cos\theta + \frac{u_z}{v_{ti}}\right)^2, \quad (13)$$
$$\chi_{13} = -\left(\frac{\omega_{pi}}{\omega}\right)^2 \eta_i Z[\eta_i] \sin\theta\cos\theta + 2\frac{\omega_{pi}^2}{\omega^2}(1+\eta_i Z[\eta_i]) \left(\eta_i \sin\theta + \frac{u_x}{v_{ti}}\right)\left(\eta_i \cos\theta + \frac{u_z}{v_{ti}}\right), \quad (14)$$
$$\eta_i = \frac{\omega - k_x u_x - k_z u_z}{v_{ti} k}. \quad (15)$$

Equations (1) through (15) define the desired dispersion relation whose solution will be subject of the following subsection.

3 Solution

As an initial step, we introduce dimensionless quantities according to

$$\omega = \Omega_i \tilde{\omega}, \quad \mathbf{k} = \frac{\Omega_i}{v_A}\tilde{\mathbf{k}}, \quad \mathbf{u} = v_A \tilde{\mathbf{u}}, \tau = \frac{\omega_{pe}^2}{\Omega_e^2}, \quad \mu = \frac{m_i}{m_e}. \tag{16}$$

Throughout the whole numerical analysis the following input parameters remain fixed [Matsukiyo and Scholer, 2003]

$$\mu = 1826, \quad \tau = 2 \times 10^4, \quad \tilde{\mathbf{u}} = 2. \tag{17}$$

These values are chosen to allow a comparison between our results and that from [Matsukiyo and Scholer, 2003] and correspond to physical parameters at the foot of the Earth's bow shock.

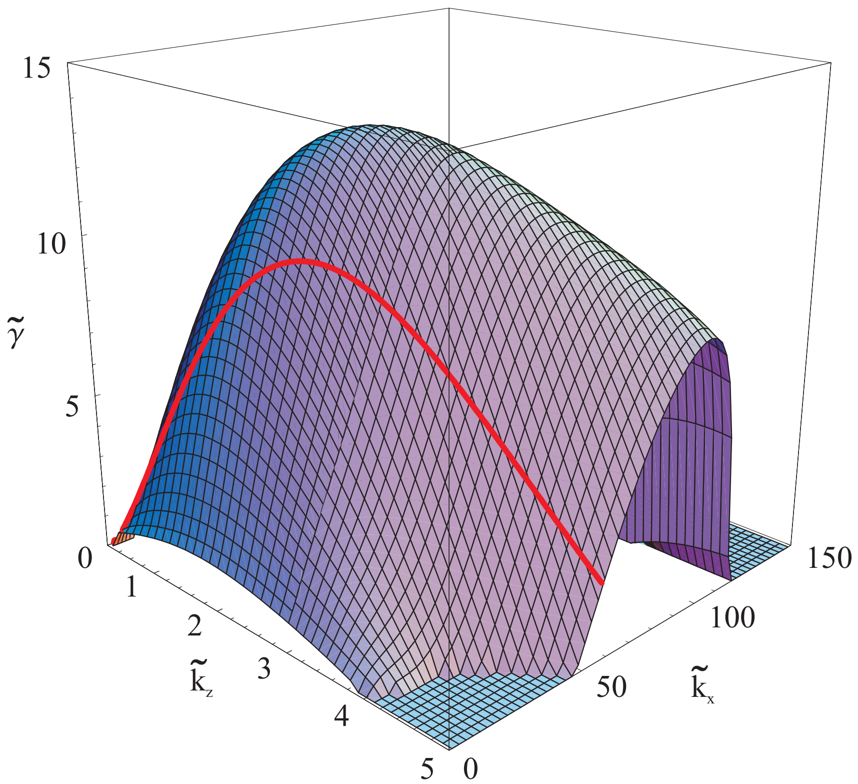

Figure 1: Growth rate as a function of the wave vector in normalized units.

We apply a numerical procedure to obtain the zeroes of the dispersion relation. An example is shown in Figure 1 where the normalized growth rate $\tilde{\gamma}$ is shown as a function of $\tilde{k}_x$ and $\tilde{k}_z$. Here, the electrons and ions are assumed to be of the same temperature, the plasma beta is equal to 0.25, and the angle α is taken to be 85°. The red line in this plot corresponds to results of [Matsukiyo and Scholer, 2003], who considered the special case of parallel ion bulk flow and wave vector. The maximum growth rate of the MTSI occurs nearly at the direction perpendicular to the magnetic field. For exactly perpendicular propagation the MTSI disappears, whereas we have a finite growth rate for

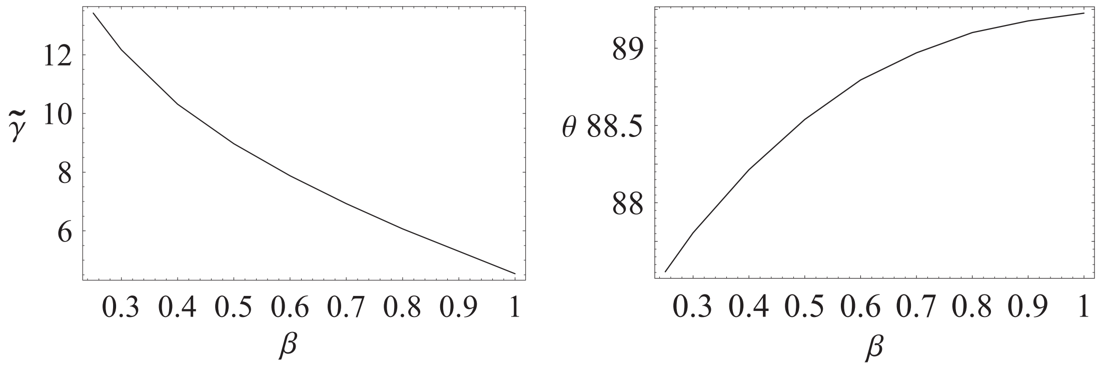

Figure 2: Maximum growth rate (left) and the corresponding location (right) as a function of the plasma beta.

parallel propagation. This limiting instability is called ion–Weibel instability [Chang et al., 1990].

As a next step, we turn our attention towards Figure 2. The left panel in this Figure shows the maximum growth rate as a function of the plasma β, which is taken to be equal for electrons and ions. The angle between the bulk flow and the magnetic field is 85°. We see that the finite temperature effectively reduce the maximum growth rate [Lemons and Gary, 1977]. Thus, as the plasma gets warmer the efficiency of MTSI is reduced. The right panel in Figure 2 demonstrates the influence of the plasma beta on the location of the maximum growth rate measured in terms of the angle θ. For all values of the plasma beta the maximum growth rate occurs nearly perpendicular the magnetic field. However, for smaller values of β the maximum growth rate is shifted farer away from the exact perpendicular direction.

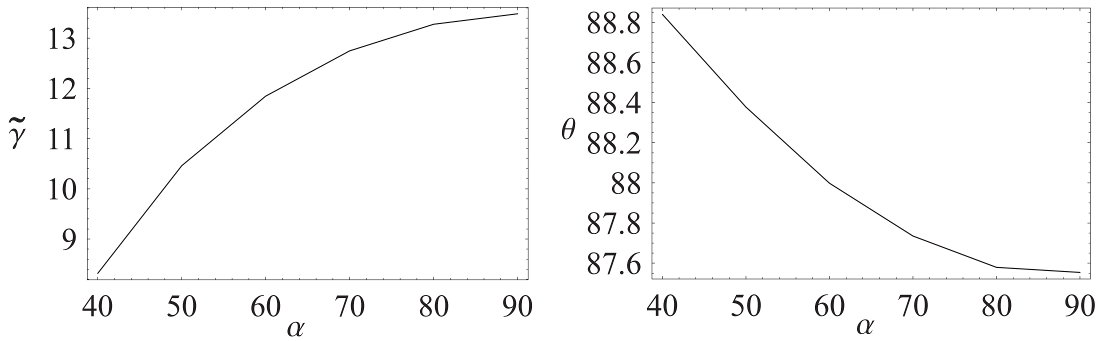

Figure 3: Maximum growth rate as a function of the angle between the flow direction of the ion contribution and the background magnetic field.

Now, we consider the effect of the direction of the bulk flow of the ions on the maximum growth rate of the MTSI as shown in Figure 3. For this Figure, the plasma beta is equal to 0.25 and the flow direction of the ions varies between $\alpha = 90°$ and $\alpha = 40°$. We see that the maximum growth rate of the MTSI decreases as the angle between the magnetic field and bulk flow becomes smaller. We note that the Weibel instability does not show such a behavior, it is unaffected by the variation of the quantity α. The right panel in Figure 3 shows the location of the maximum growth rate as a function of the angle α.

Again, the maximum growth rate is located approximately in the perpendicular direction to the magnetic field, however, is shifted farer away for larger values of α.

4 Summary

In this paper we study the MTSI in a two component plasma by applying the traditional approach of unmagnetized ions and magnetized electrons. A coordinate system is adopted in which the electrons remain stationary and the ions move across the magnetic field. In contrast to previous studies in this field, we allow the bulk flow of the ions to be inclined towards the direction of the background magnetic field under the angle α. We use the linearized Vlasov electromagnetic dispersion relation to study the influence of various plasma parameters on the growth rate of the MTSI. These parameters include the plasma beta and the angle between the flow direction of the ions and the magnetic field. The main result is that the maximum growth rate of the MTSI decreases as the angle between the magnetic field and flow direction of the ions becomes smaller. From the physical point of view this circumstance can be explained as follows. In case of wave propagation across the magnetic field, there is no Landau damping due to the electron contribution, because they are magnetized and do not have a drift across the magnetic field. But, in the case of oblique wave propagation there appears to be a k_z–component along the magnetic field and there can be Landau damping due to the resonant electrons moving along the magnetic field. In this case, the maximum growth rate decreases due to this Landau damping.

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