ON THE 3D KINETIC MODELING OF A MAGNETOTAIL/SOLAR STREAMER BY A PLASMA FLOW OVER MAGNETIC DIPOLE AND TOROID

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Abstract

The problem of magnetotail (or solar streamer) 3D structures formed in the solar wind (SW) flow is solved in terms of the Vlasov kinetic approach. The external source of the magnetic field, presented as a superposition of a magnetic dipole (MD) and a magnetic toroid (MT), is considered inside the SW flow. The SW is a hot, collision-less plasma with a maxwellian velocity distribution function. We assume that the wind particles are not trapped by the magnetic field of the 3D structure and are only slightly disturbed. Thus, a linear approach can be applied to the problem. The generated 3D magnetosphere-like structure is formed by two different types of magnetic field cylindrical harmonics: the line-current and the solenoidal ones. The line-current harmonic has the form of two opposite line-currents running along the flow. It is generated by the MD component of the magnetic source. The solenoidal harmonic is associated with the "theta-type" current structure with the "neutral sheet" formed as a superposition of two solenoids. It appears due to the MT component of the external magnetic source. The kinetic energy of the SW flow is dissipated by the Landau damping mechanism connected with inertia of particles which are under resonance condition with the magnetic field. Both types of harmonics are expressed via a special characteristic function parameterized in terms of the magnetic Reynolds number depending on the specific plasma dispersion scale. Thereby takes place a slow power law asymptotic decay of the characteristic function in the downward direction of the plasma flow and its fast aperiodic decrease with a spatial modulation in the direction perpendicular to the flow. This results in a faster spatial decay of the theta-type configuration as compared to the line-current one.

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1 Introduction

Chapman and Ferraro were the first in plasma physics who considered in 1931 the problem of the interaction of a source of magnetic flux with a plasma flow. In further research the process of inductive interaction of the source of magnetic flux with a moving plasma cloud forms the physical backgrounds to study the formation of the Earth magnetosphere and the streamer belt in the Solar corona. The solution of this problem has a long history and different approaches were applied [Siscoe, 2001], and they reflected partly the change of the observing position from a location at the source to the outer space. Our task here is to create the background for kinetic 3D solutions of this problem in the basis of exact analytical solutions in hot collisionles flowing plasmas. On this way, some former MHD results can be reconsidered. In our approach to the problem the observation position is located in the outer space, where far regions of the magnetotail/solar streamer are formed (Fig.1, Fig.2). Initial steps in this direction can be found in the paper [Gubchenko, 1988].

A magnetic flux source is equivalent to the presence of an eddy current $\mathbf{j}_t$ in the plasma flow. The source $\mathbf{j}_t$ can be treated like a sum of external currents $\mathbf{j}_{ext}$ and by the currents of plasma particles trapped by magnetic field $\mathbf{j}_{trap}$ with its own complicated physics without producing a direct flow. The plasma flow $n_0 v'_x$ having velocity $\mathbf{v}' = v'_x \mathbf{x}_0$ is associated with the Solar Wind (or radial expansion of the solar corona) and can be characterized by plasma frequency $\omega^2_{p\alpha}$ and thermal velocity v_α for electrons $\alpha = e$ an ions $\alpha = i$ and by some velocity distribution function (VDF) before the interaction with the source.

A finite effective resistivity and diamagnetic properties of the flow cause changes of the topology of the initial magnetic field to a self consistent structure with plasma flow and 3D electromagnetic tail/wake structures having inside three types of elements: current sheets, magnetic islands, and magnetic ropes with scale ΔL [Gubchenko et al., 2005]. As for magnetic islands, they can be treated like elementary CME/magnetic substorm events. The formation of the structures or even the work of the generator can be considered as a result of "blow up" from the source or as a sort of the source projection down to the flow of the field. Subsequently the result strongly depends on the original angle morphology of the source at the aperture r_0. In our consideration we get typical elements like sheets and ropes only. It is not enough to have only magnetic dipole component (MD) in the source configuration, and, to be more adequate to reality, we add into the consideration a toroidal component (MT) to the source. The ratio of the MT component to MD component in the source we define by parameter $\Gamma_{\tau\mu} = I_\tau / I_\mu$ where I_τ is the full current in the MT component and I_μ is the full current in the MD component.

The scales $\Delta L(\mathbf{X}, r_0, r_{disp})$ of 3D structures are defined by a form factor r_0 of the source $\mathbf{j}_t$, by the position $\mathbf{X}$ relative to the source and by internal dispersion scales of the space plasma itself r_{disp}. Our purpose is to connect observed scales and the morphology of 3D structures with the source form factor and dispersion scales, originating from time and spatial dispersion properties of the hot collisionless flowing plasma.

The form factor of the source is defined via the radial spatial scale r_0 and the character of the angle distribution of the current $\mathbf{j}_t(r, \theta, \varphi)$. The character of the angle distribution in the source defines types of structural elements "blown" out from the source. We limit ourself to the simplest models for the eddy current distribution and postulate it by the sum of the first dipole (poloidal) and the first toroidal (anapole) spherical harmonics. These harmonics are characterized by vectors $\vec{\mu}$ and $\vec{\tau}$. We study the source in the case $\vec{\mu}\perp\vec{\tau}$ which introduces initial asymmetry on the magnetic field vacuum structure with some compression of the dipole magnetic field lines in front of the source and formation of a decompression of magnetic field lines up to the level for the formation of a "neutral point" behind the source. The situation most typical to magnetosphere and solar streamer analysis and has a big initial demand. Other typical situation which we not consider here but which can be also easily analyzed by the below proposed characteristic function M_G is $\vec{\mu}\|\vec{\tau}$. The last situation describes current-carrying magnetic loops on the Sun surface with spiralling (shear) current and magnetic field. It is evident that in reality both cases appear in superposition.

For a cold ideal MHD plasma, we get a time dispersion skin scale $c/\omega_{p\alpha}$, which defines in particular the fine structure of travelling nonlinear MHD waves. For hot plasmas, treated in kinetics, we get a spatial dispersion scale like the electrostatic Debye scale r_{DE} which defines electrostatic structures of travelling ion-sonic solitons. We can operate with $r_{c\alpha}$ being the gyroradius of thermal particle in magnetized plasmas. The scales above are small compared to the observed large scales ΔL of a space plasma resting e.m. structures and can not be applied directly. Observed scales $\Delta L(X, r_{disp})$ are connected with electromagnetic structures under study which are associated with the mode of inductive fields in a plasma having large r_{disp} and we note that far from the source $\Delta L >> r_{disp}$. The inductive mode can produce structures based on scale r_{disp} both like dissipative (resistive) $r_{disp} = r_G$, and nondissipative (diamagnetic) $r_{disp} = r_{DM}$, and we get a ratio $G = r_G/r_{DM}$ which we call "Magnetosphere Quality". Here we used analogy with the Quality of oscillators. The Quality depends on the form factor of the plasma velocity distribution function (VDF) of the incoming plasma flow.

It is important to note that all basic scales of the plasma appear in linear consideration. It is our task to get new large electromagnetic scales and illustrate the importance of new dispersion scales in the particular problem here. The new scales r_{disp} are the background to develop the model up to get nonlinear solutions.

We study the problem in terms of the Vlasov/Maxwell equations in the limiting Vlasov/Darwin approximation. The plasma in the flow is assumed to be hot, collisionless, and we have "fly by" particles which are non magnetized by the external field ($\mathbf{B}_0 = 0$) and the field of the plasma $\mathbf{B} = \nabla \times \mathbf{A}$. The plasma has a maxwellian velocity distribution function (VDF) at infinity in front of the source $f_\alpha = f_{M\alpha}$ and it transforms to 6D nonmaxwellian VDF $f = f_M + f_1$ which is selfconsistent with the excited 3D electromagnetic fields $\mathbf{A}(\mathbf{X}, t)$. Plasma kinetic effects are important in the considered situation, $v' << v_e, v_i$ (solar corona expansion), $v_i << v' << v_e$ (solar wind at the Earth orbit). Here we have different types of interaction of particles/field in the VDF. The limit $v' >> v_e, v_i$ is treated mainly in cold MHD where kinetic effects are not important.

We get particles which are "trapped" by the magnetic field and we postulate their electromagnetic action on "fly by" particles by parameterizing their current in the source by MD and MT. Two types of "fly by" particles, resonant-accelerated and nonresonant-adiabatic ones, can be found in the VDF in the kinetic regime on linear approach. Resonant particles have no velocity relative to magnetic field and they are responsible for the dissipation effect forming a large scale anomalous skin depth r_G. Nonresonant particles produce diamagnetic properties of the flow forming a large scale magnetic Debye scale r_{DM}. The ratio $G = r_G/r_{DM}$ defines the resistive or diamagnetic nature of the formed tail/streamer. Keeping in the analytic model the basic kinetic effects in magnetic flux-plasma flow interaction, we try here to find the morphological similarity with the observed 3D structures at the Sun corona and the geomagnetic tail (Fig.1, Fig.2).

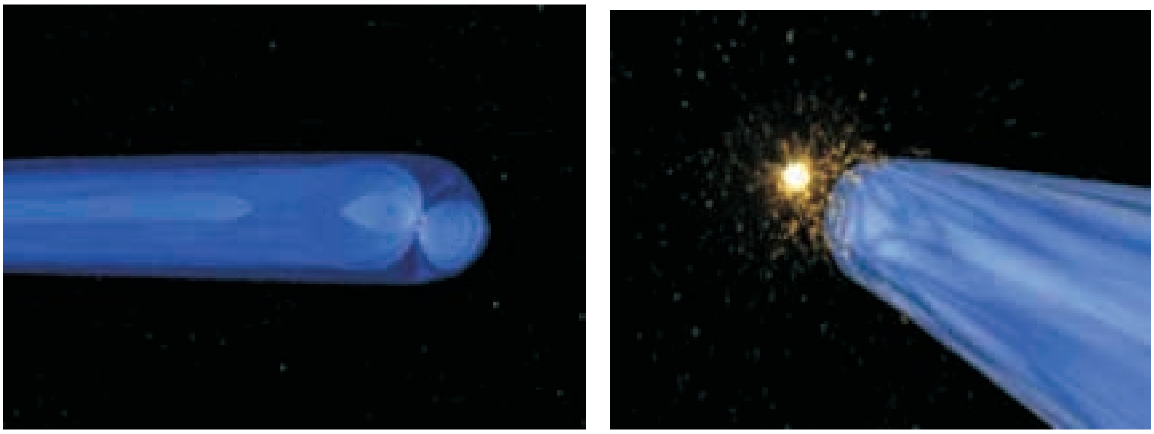

Figure 1: Schematic view of the 3D Earth magnetosphere tail formed by solar wind flow in two projections: morning/evening (left) and night (right) (Courtesy of NASA). A near tail is formed by neutral sheets and magnetic islands and a far tail is formed by magnetic ropes.

The paper is organised as follows. In the next section we discus in more detail the structure of the magnetic flux source. In the following section we get general expression for the electromagnetic field. In the fourth section we describe the characteristic function of the problem. The fifth part is devoted to a discussion of the streamer/tail in terms of cylindrical harmonics.

2 External magnetic source

It is convenient to solve the problem in a reference frame $\mathbf{x}$ connected to a resting maxwellian plasma so that the source of magnetic flux $\mathbf{j}_t(\mathbf{x}, t)$ is in direct relative motion with the velocity $v'\mathbf{x}_0$. In the reference frame of the source $\mathbf{X}$, we get $\mathbf{j}_t(\mathbf{X})$, where $\mathbf{X} = \mathbf{x} - v'\mathbf{x}_0 t$.

The external source of the magnetic field is a superposition of a magnetic dipole (MD) and a magnetic toroid (MT) which have densities $\vec{\mu}$ and $\vec{\tau}$; higher order spherical harmonics forming the source are ignored but nevertheless in part illustrated in Fig.3. The eddy current for the magnetic source has the following expression:

$$\mathbf{j}_t = \mathbf{j}_\mu + \mathbf{j}_\tau + ... = c[\nabla \times \vec{\mu}] + c[\nabla \times [\nabla \times \vec{\tau}] + ..., \tag{1}$$

where for moving MD ($\vec{\mu}$) and moving MT ($\vec{\tau}$) we have the following formulas and figure (Fig.3),

$$\vec{\mu} = \mathbf{z}_0 \mu_0 \frac{1}{(2\pi r_0^2)^{3/2}} \exp(-\frac{\mathbf{X}^2}{2r_0^2}), \tag{2}$$

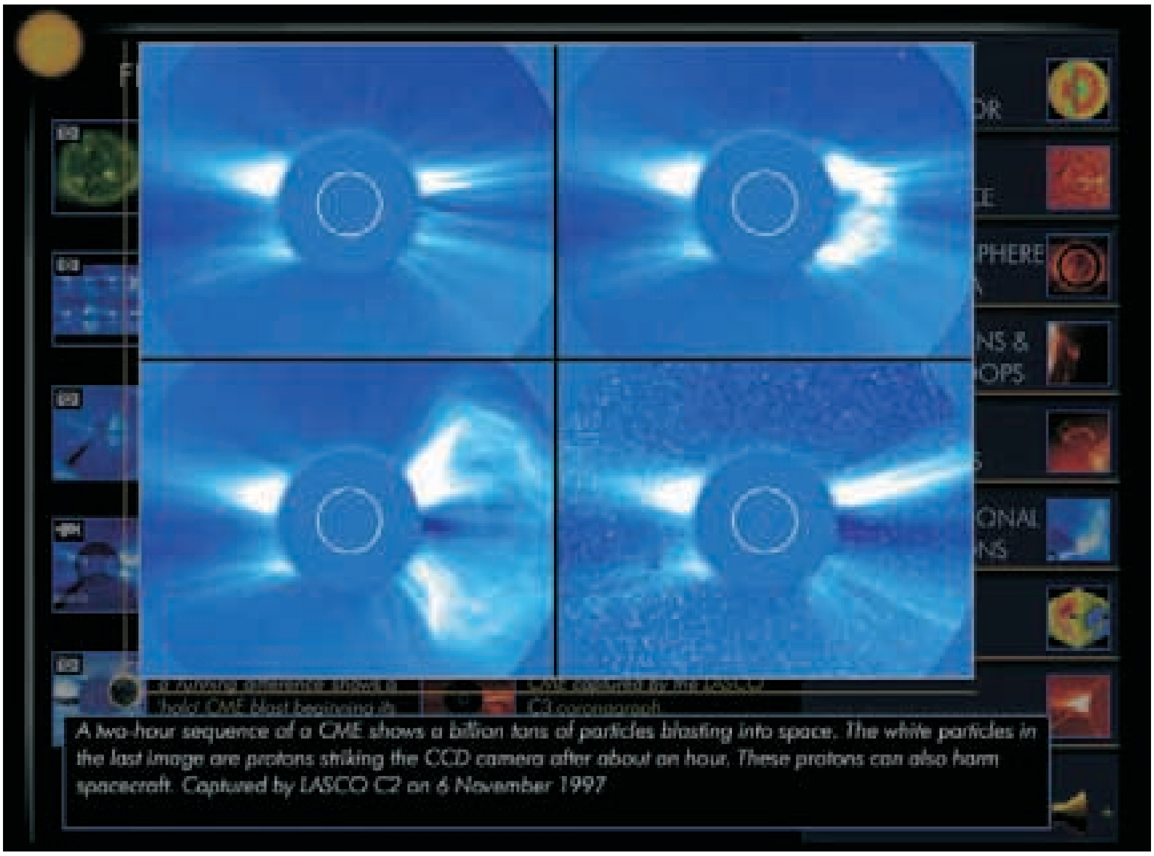

Figure 2: 3D streamer belt formed over the magnetoactive regions by the solar corona expansion. Helmet structures and CME are associated with magnetic islands in the heliospheric sheet. Ray structures are associated with magnetic ropes.(Courtesy of NASA)

$$\vec{\tau} = \mathbf{y}_0 \tau_0 \frac{1}{(2\pi r_0^2)^{3/2}} \exp(-\frac{\mathbf{X}^2}{2r_0^2}). \quad (3)$$

We can express the integral values of magnetic moment μ_0 and magnetic toroidal moment τ_0 of the source via integral currents I_μ and I_τ in their current systems. We get

$$\mu_0 = I_\mu \pi r_0^2, \quad (4)$$

$$\tau_0 = I_\tau \frac{4}{3} \pi r_0^3. \quad (5)$$

The current I_μ is formed by "ring" currents produced partly by the external dipole and partly by current from particles in magnetic trap. This current is distributed on the characteristic surface $S_0 = \pi r_0^2$. The current I_τ is formed by current of "circular solenoid" and produced by particles in magnetic trap. This current is distributed in the characteristic volume $V_0 = (4/3)\pi r_0^3$. The dimensionless parameter $\Gamma_{\tau\mu} = I_\tau / I_\mu$ characterizes the weight of toroidal harmonics versus dipole harmonics in the magnetosphere. In the case $\Gamma_{\tau\mu} = I_\tau / I_\mu << 1$, we have a dipole type magnetosphere and in the case $\Gamma_{\tau\mu} = I_\tau / I_\mu >> 1$ we have a toroidal type magnetosphere. The calculation of the introduced parameter $\Gamma_{\tau\mu}$ is subject to nonlinear considerations of the problem. We can get estimation of the value

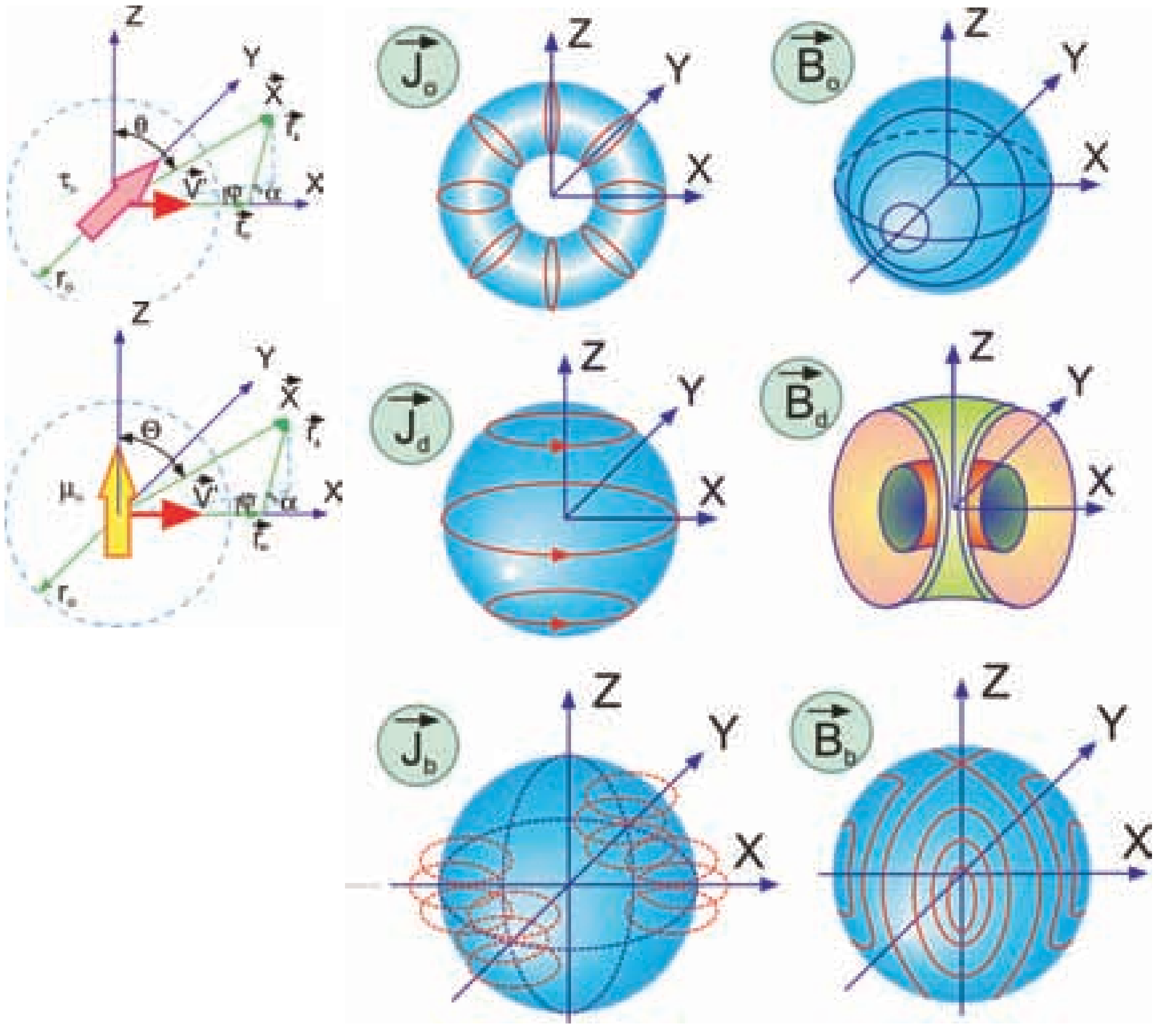

Figure 3: Structure of the source of magnetic flux which is distributed on the radial scale r_0. The first column represents the orientation of the sources and coordinate system $\mathbf{X}$ in spherical and cylindrical representations, the second column is the current distribution for the toroid $\mathbf{j}_0$, for the dipole $\mathbf{j}_d$ and it is $\mathbf{j_b}$ for the double toroid as an example of higher order multipoles. In the third column are pictures of magnetic field lines. In the first raw we have magnetic toroid which produces ring magnetic field lines in the plane $Z - X$ and located on the sphere. In the second raw we have the magnetic dipole with ring currents in the plane $X - Y$. In the third raw we have the illustration of the higher order spherical multipole - double toroid which is not included in our further considerations.

$\Gamma_{\tau\mu}$ via observations of near regions of the Earth magnetosphere ($\vec{\tau} \cdot \vec{\mu} = 0$) or study magnetic loops over active region of the Sun where the more common case with a spiralling current can be realized ($\vec{\tau} \cdot \vec{\mu} \neq 0$).

3 General relations for generated electromagnetic fields

Due to additional variables, 3D problems in kinetics are much more complicated as compared to the same problems in MHD. The 3D problem is studied mainly numerically and can be analytically considered only after certain simplification and introduction of new parameters G, $\Gamma_{\tau\mu}$, Re_m which keep, at the same time, basic kinetic effects connected with self-consistent interaction of plasma particles and electromagnetic field.

We distinguish two basic types of particles in the expanding solar corona (i.e., solar wind) which interacts with a magnetic field of an active region. These are trapped and untrapped or fly by particles. Electric currents created by the trapped particles are included in definition of the magnetic source, which is characterized by the source current $\mathbf{j}_t$. Since, contrary to the case of a 2D trap, in the 3D trap there is no ideal separatrix between trapped and untrapped particles, the above assumption appears as an idealization. In principle, trapped particles have their own rich physics, but for the considered model it is important that they do not create a plasma flow relative to the source. We study here selfconsitently perturbations of the direct motion of untrapped plasma particles characterized by the perturbed VDF, $f_{M\alpha} + f_{1\alpha}$. These perturbations are self-consistent with the electromagnetic field $\mathbf{A}$ related to the moving (relative motion) magnetic source $\mathbf{j}_t$.

Assuming that the direct motion of non-trapped particles $\mathbf{v}_0 = const$ with different constants in the solar wind having the VDF $f_M(\mathbf{v}_0)$ is only slightly disturbed ($\mathbf{v} = \mathbf{v}_0+\mathbf{v}_1+...$) by the field $\mathbf{A}$, and that there are no nonlinear effect of magnetization ($kr_{c\alpha} >> 1$), we treat the flow part of the plasma within a linear approach, i.e., we suppose $f_\alpha(\mathbf{v}) = f_{M\alpha} + f_{\alpha 1}$. Here, $r_{c\alpha}$ is spatial scale of possible particle periodic motions in the magnetic field and k^{-1} is the scale of plasma fields. Perturbation of VDF, f_1, which results in a current $\mathbf{j}_1 = \Sigma_\alpha e_\alpha \int d\mathbf{v}\mathbf{v} f_{\alpha 1}$, may be expressed via potentials $\mathbf{A}$ and φ and the dielectric tensor ε_{ij}. By this, the vector potential is presented as a sum of two components: $\mathbf{A} = \mathbf{A}_\mu + \mathbf{A}_\tau$, where $\mathbf{A}_\mu$ and $\mathbf{A}_\tau$ are generated by the MD and MT parts of the source current $\mathbf{j}_t = \mathbf{j}_\mu + \mathbf{j}_\tau$, and expressed via the transversal component ε_t of the plasma permittivity tensor $\varepsilon_{ij}(\omega, \mathbf{k})$ in which all effects of the field - plasma interaction are concentrated.

According to the condition of a weak magnetic field in the streamer formation region $kr_c >> 1$, we are dealing with an isotropic plasma which is not magnetized by the field of streamer. In this case, in the rest frame of reference of unperturbed moving plasma described by $f_M(\mathbf{v} - \mathbf{v}')$, we can describe the media by the simple diagonal tensor $\varepsilon_{ij}(\omega, \mathbf{k}) = (\delta_{ij} - \frac{k_i k_j}{k^2})\varepsilon_t + \frac{k_i k_j}{k^2}\varepsilon_l$ with components $\varepsilon_{t1} = \varepsilon_{t2} = \varepsilon_t$ and ε_l.

The stationary electromagnetic field produced in a moving plasma by the streamer related magnetic source is given by [Gubchenko, 1988]

$$\mathbf{A} = \int d\mathbf{k} \mathbf{A}_\mathbf{k} \exp(i\mathbf{k}\mathbf{X}), \tag{6}$$

with

$$\mathbf{A}_\mathbf{k} = \frac{4\pi}{c} \frac{\mathbf{j}_{t\mathbf{k}}}{k^2 D_T(\mathbf{k}, \mathbf{k}\mathbf{v}')}, \tag{7}$$

where $\mathbf{j}_{t\mathbf{k}} = \mathbf{j}_{\mathbf{k}\mu} + \mathbf{j}_{\mathbf{k}\tau}$,

$$\mathbf{j}_{\mathbf{k}\mu} = -\frac{ic\mu_0}{(2\pi)^3} \exp(-\frac{k^2 r_0^2}{2})(k_y \mathbf{x}_0 - k_x \mathbf{y}_0), \tag{8}$$

and

$$\mathbf{j}_{\mathbf{k}\tau} = \frac{c\tau_0}{(2\pi)^3} \exp(-\frac{k^2 r_0^2}{2})(\mathbf{k} k_y - k^2 \mathbf{y}_0). \tag{9}$$

Here $\mathbf{j}_{\mathbf{k}\mu}$ and $\mathbf{j}_{\mathbf{k}\tau}$ are Fourier harmonics of the MD and MT components of the current in the magnetic flux source. The effects of the plasma are characterized by $D_T(\mathbf{k}, \mathbf{kv}') = 1 - \frac{(\mathbf{kv}')^2}{(ck)^2}\varepsilon_t(\mathbf{kv}', \mathbf{k})$. By this $D_T(\mathbf{k}, \omega) = 0$ is the dispersion equation of plasma. In the case of vacuum in the quasistatic region ($\omega/ck <<< 1$) it turns $D_T = 1$.

Moving in plasma magnetic flux sources $\mathbf{j}_t$, besides of the non-potential electromagnetic field ($\mathbf{A}$) excite also a potential electrostatic field (φ), but this potential fields are not considered here.

According to Eq. (6), coronal streamer is "laminar" flow and its formation can be treated as Cherenkov "radiation" of the non-propagating inductive plasma mode $\omega_r(\mathbf{k}) = 0$, $\omega_i(\mathbf{k}) \neq 0$ formed by "coherent" harmonics $\mathbf{A}_\mathbf{k}$ generated in plasma by MD and MT moving sources. The analogy for Mach cone in such non-propagating mode "radiation" case is degenerated into a wake/tail structure of the 3D streamer magnetosphere.

In the case of Maxwellian plasma ($f = f_M$), parameterized only by the plasma frequency $\omega_{p\alpha}$ and thermal velocity of particles v_α, we obtain

$$\varepsilon_t(\omega, \mathbf{k}) = 1 + \Sigma_\alpha \frac{\omega_{p\alpha}^2}{\omega^2} i\pi^{1/2}\xi_\alpha w(\xi_\alpha). \tag{10}$$

Here $w(\xi_\alpha)$ is the Cramp function which in the regime $\xi_\alpha = \omega/kv_\alpha << 1$ has the following representation; $w(\xi) = 1 + 2i\xi/\pi^{1/2} + \ldots$. The real part of ε_t is responsible for the polarization of the plasma under the action of electric/magnetic field, whereas the imaginary part contains the dissipation effects. For our case, the dissipation is connected with the Landau damping effect of the inductive electromagnetic mode.

For the slow motion, i.e. when $|\mathbf{v}'| << v_\alpha$, we have $\xi_\alpha = \mathbf{kv}'/|k|v_\alpha << 1$. This yields

$$D_T(\mathbf{k}, \mathbf{kv}') = 1 - \frac{(\mathbf{kv}')^2}{(ck)^2}\varepsilon_t(\mathbf{kv}', \mathbf{k}) \approx 1 - \frac{i}{k^2 r_G^2}\frac{k_x}{|\mathbf{k}|} + \frac{1}{k^2 r_{DM}^2}. \tag{11}$$

In this equation, the spatial dispersion scale r_{disp} appears as two different scaling parameters;

$$r_G^{-2} = \sum_\alpha \frac{\omega_{p\alpha}^2}{c^2}\frac{v'}{v_\alpha}, \tag{12}$$

which is the anomalous skin scale for field in a moving collisionless hot plasma, and

$$r_{DM}^{-2} = \sum_\alpha \frac{\omega_{p\alpha}^2}{c^2}\frac{v'^2}{v_\alpha^2}, \tag{13}$$

which is the magnetic Debye scale due to plasma anisotropy $\kappa_\alpha = v'^2/v_\alpha^2$ [Gubchenko et al., 2003]. Thus, in the case of slow relative motion of plasma and magnetic source, in the rest frame of the source the plasma shows also diamagnetic properties.

The value r_G is connected to the presence of accelerated particles which are in resonance conditions and have no motion relative to the magnetic field. They are mainly accelerated by the eddy electric field $\mathbf{E} = -(1/c)\partial\mathbf{A}/\partial t$. These particles are responsible for the plasma resistivity. The scale r_G is connected with the anomalous skin scale r_a, which describes penetration of the oscillatory field into a hot restive Maxwellian plasma [Lifshitz and Pitaevskii, 1981]. According to the above expression for D_T, the r_G-screening effect is anisotropic. As for r_{DM}, this spatial dispersion scale is connected to the presence of moving nonresonant particles, which are under the action of the magnetic field $\mathbf{B} = \nabla \times \mathbf{A}$. This effect is isotropic and it is similar to the effect of isotropic electrostatic Debye screening on the scale $r_{DE}^{-2} = \Sigma_\alpha \omega_{p\alpha}^2 / v_\alpha^2$.

We can introduce also a dimensionless plasma parameter,

$$Re_m = \frac{r_0}{r_G}, \tag{14}$$

which has the meaning of a magnetic Reynolds number in the Vlasov plasma. In the case of $Re_m >> 1$ we are close to the "ideally conductive" plasma and the original (vacuum) magnetic field of the source $\mathbf{j}_t$ is strongly disturbed by eddy plasma currents excited near the source. The case of $Re_m << 1$ corresponds to a non-conductive media, when the original (vacuum) field of the source is only slightly disturbed by the plasma currents near the source.

The dimensionless plasma parameter,

$$D_M = \frac{r_0}{r_{DM}}, \tag{15}$$

has the meaning of a magnetic Debye number [Gubchenko et al., 2003]. The ratio $G = D_M/Re_m$ characterizes the relative role of diamagnetic and resistive effects in the plasma. It may be considered as a kind of a quality parameter of the system.

In the directly flowing solar wind Maxwellian plasma considered here, we have $r_G << r_{DM}$ ($G << 1$) and the 3D structures, which we get are resistive and non-diamagnetic. The ratio G can be radically changed in non-Maxwellian space plasmas. This effect is considered by Gubchenko et al., [2004] for plasmas with an anisotropy namely, temperature anisotropy, double plasma flows, and electric currents.

4 Characteristic function of the problem

According to Eq. (6), it is convenient to express streamer fields generated by the MD and MT source components, i.e., by $\mathbf{j}_\mu$ and $\mathbf{j}_\tau$, via derivatives of a characteristic function

$$M_G(\mathbf{X}, Re_m, G) = \frac{4\pi}{(2\pi)^3} \int d\mathbf{k} \frac{\exp(-\frac{k^2 r_0^2}{2} + i\mathbf{kX})}{k^2 D_T(\mathbf{k}, \mathbf{kv}')} \tag{16}$$

These will have the following form;

$$\mathbf{A}_\mu = \mu_0 (\frac{\partial M_G}{\partial Y} \mathbf{x}_0 - \frac{\partial M_G}{\partial X} \mathbf{y}_0), \tag{17}$$

$$\mathbf{A}_\tau = \tau_0 (\frac{\partial^2 M_G}{\partial X \partial Y} \mathbf{x}_0 + (\frac{\partial^2 M_G}{\partial X^2} + \frac{\partial^2 M_G}{\partial Z^2}) \mathbf{y}_0 - \frac{\partial^2 M_G}{\partial Z \partial Y} \mathbf{z}_0). \tag{18}$$

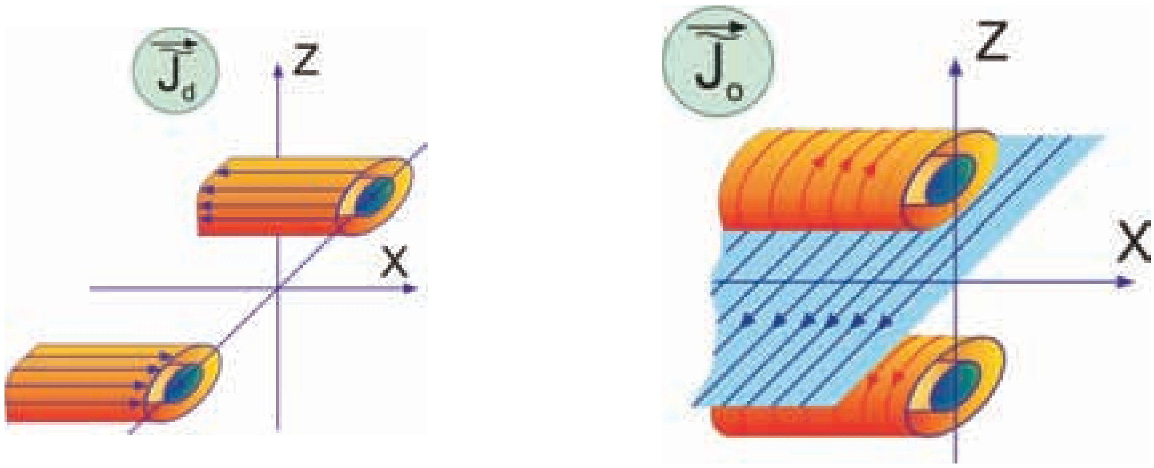

Figure 4: Left: Two wire-current system forming cylindrical dipole. Direct currents are in opposite directions. These currents provide "normal component" B_n of magnetic field. Right: The current system of the solenoidal harmonic formed by two solenoids with a "neutral current sheet" between them.

Under the integral in Eq.(16) we have a non-analytical function D_T which causes during the integration the appearance of cuts and poles in the complex integration k-planes. The function $M_G(\mathbf{X}, Re_m, G)$ can be considered as a Green function in the limit $r_0 \to 0$. In the vacuum case ($D_T = 1$) for $r_0 \to 0$, we obtain an isotropic function $M_V = 2/\pi r$ with a power-low $\sim r^{-1}$ decrease with distance to the source.

According to Eqs. (17) and (18) the MT part of the field, i.e. $\mathbf{A}_\tau$, has higher order derivatives of the function M_G then the MD part. This means a faster decay of the MT part of the field as compared to the MD part. Thus, far from the source ($r \to \infty$) for $\Gamma_{\tau\mu} \sim 1$, mainly the $\mathbf{A}_\mu$ part contributes to the field. If $\Gamma_{\tau\mu} >> 1$, the fields in far regions of the streamer are defined via MT component of the source. Near the source we see both, components created by the MD (i.e. $\mathbf{A}_\mu$), and MT (i.e. $\mathbf{A}_\tau$) parts of the source.

In the general case, the function M_G is asymmetric and has a power law decrease to infinity with different values of the power index depending on the angle formed by the radius-vector of a given point and the flow velocity $\mathbf{v}'$ vector. In the front part ($X > 0$) it decreases faster then M_V does and forms the long tail/streamer part for $X < 0$ with a slower decrease.

The regions near the source $\mathbf{j}_t$ and far from the source can be studied by analytical methods. Inside the source $r = 0$, an asymptotic (Taylor-type) expansion of the function M_G is possible. It gives information on influence of electric currents in the plasma flow on the magnetic field redistribution inside the source. Another approximation is connected with the representation of $\exp(i\mathbf{kX})$ under the integral in Eq.(16) by spherical harmonics, using spherical coordinates r, θ, ϕ. Such an approach produces an infinite set of dipole and toroidal type spherical harmonics in the 3D magnetic field approximation. These harmonics decay not slower then $\sim r^{-3}$. This means fast skinning of the original field [Gubchenko, 1988]. This spherical approach is, however, not effective for the analysis of fields in the streamer region.

For the analysis of fields in the elongated narrow streamer region, it is convenient to use cylindrical coordinates; X, $r_\perp = (Y^2+Z^2)^{1/2}$, α'. The analogous coordinate system in k-space will be; $k_x = k_\|$, $k_\perp = (k_y^2+k_z^2)^{1/2}$, α''. After integration in Eq. (16) over the angle α'' in $\mathbf{k}$-space taking into account the representation $\exp(i\mathbf{k}_\perp \mathbf{X}_\perp) = \exp[ik_\perp r_\perp \cos(\alpha' - \alpha'')] = \Sigma_{l=-\infty}^{l=\infty} J_l(k_\perp r_\perp)\exp[il(\pi/2 - \alpha' + \alpha'')]$, the characteristic function M_G in the case $G = 0$ may be written in a more simple 2D form:

$$M(\chi, \rho_\perp, Re_m) = \frac{1}{\pi r_G}\int_0^\infty d\xi_\perp \xi_\perp J_0(\xi_\perp \rho_\perp)\exp(-\frac{\xi_\perp^2 Re_m^2}{2}) I_x(\xi_\perp, \chi, Re_m), \tag{19}$$

with

$$I_x(\xi_\perp, \chi, Re_m) = 2Re\int_0^\infty d\xi_x \exp(i\xi_x\chi - \frac{\xi_x^2 Re_m^2}{2})\frac{(\xi_\perp^2+\xi_x^2)^{1/2}}{(\xi_\perp^2+\xi_x^2)^{3/2} - i\xi_x}, \tag{20}$$

where $\xi_\perp = k_\perp r_G$, $\xi_x = \xi_\| = k_x r_G$ are real dimensionless wave vector components and they both are positive values in integrals after transformation of the original integral (16). We introduced dimensionless coordinates $\chi = X/r_G$, $\rho_\perp = r_\perp/r_G$, where $\chi < 0$ is the space occupied by the tail/streamer, and $J_0(\xi_\perp\rho_\perp)$ is the Bessel function.

The Eqs. (19) and (20) indicate cylindrical symmetry of the function M_G. By this, its behavior is influenced by only one dimension-less parameter, Re_m. This relatively simple 2D form of the function M_G is connected with the simple form of dielectric tensor ε_{ij} of the non-magnetized Maxwellian plasma. It shows that the whole problem is basically only a 2D one. Therefore, all the 3D effects in the field and flow structure of the streamer are only related to the 3D geometry of the source $\mathbf{j}_t(X, r_\perp, \alpha')$.

An asymptotic analysis of Eqs. (19) and (20) is based on the assumption of large scale ($\Delta L >> r_G$) and narrow structure of the far regions of the tail at $X \to -\infty$. For this purpose we operate with approximations of the function first under integral I_x for small $\xi_\perp << 1$ at the lower part of the complex plane ξ_x. In this approximation, we get the pole $\xi_1 \approx -i\xi_\perp^3$ close to 0 which finally defines the tail/wake at far distances. We get also a cut there which begins at $\xi_x = -i\xi_\perp$ and goes to infinity, it has small affection on the asymptotic of the function M.

After we get the asymptotic for $\chi < 0$ via a deformation of the initial integration way along the real ξ_x to the lower part of the imaginary axis ξ_x where the role of the pole ξ_1 is the strongest. We ignore also at $X \to -\infty$ the spatial structure of the source r_0. As a result, we get $I_x \approx \xi_\perp exp - \xi_\perp^3|\chi|$ and we find finally following approximation;

$$M \sim \int_0^\infty d\xi_\perp \xi_\perp^2 J_0(\xi_\perp\rho_\perp)\exp(-\xi_\perp^3|\chi|). \tag{21}$$

Near the axis of the tail, $\rho_\perp < |\chi|^{1/3}$, we use the approximation

$$J_0(\xi_\perp\rho_\perp) = \Sigma_{k=0}^{k=\infty}\frac{(-1)^k}{k!}(\frac{\xi_\perp\rho_\perp}{2})^{2k}$$

and get

$$M \sim [-\frac{1}{3|\chi|} - \frac{\Gamma(5/3)\rho_\perp^2}{48|\chi|^{5/3}} + \ldots], \tag{22}$$

with a slow expansion of the tail in transverse ($\rho_\perp > \Delta L$) direction. In this direction aperiodic modulation of the field takes place.

Far from the tail axis, $\rho_\perp > |\chi|^{1/3}$, we use the approximation

$$J_0(\xi_\perp \rho_\perp) \approx (\frac{2}{\pi \xi_\perp \rho_\perp})^{1/2} \cos(\xi_\perp \rho_\perp - \frac{\pi}{4})$$

and as a result we get Airy type function M which provides aperiodic behavior with a transition to a strong power decrease, which confirms the result of the spherical analysis,

$$M \sim [\frac{2^{1/2}}{3|\chi|} \exp(-\frac{2\rho_\perp^3}{27|\chi|})^{1/2} \cos(-\frac{2\rho_\perp^3}{27|\chi|})^{1/2} - \frac{\Gamma(5/2)}{\rho_\perp^3}]. \tag{23}$$

The spatial modulation of M produce radial fine structure in the cylinder with scale

$$\Delta L(X, r_G) = 3 r_G^{2/3} (|X|/2)^{1/3}. \tag{24}$$

This scale increases slowly with distance from the source. According to Eq.(21), the perpendicular component of magnetic field to the flow $B_n = [\nabla \times \mathbf{A}_\mu]_n$, decreases on the axis X in the case of $\rho_\perp < \Delta L$ (i.e., inside the tail) by a power-law; $\sim -5/3$.

5 Cylindrical harmonics and topology

The topology of a streamer magnetosphere is defined by function M_G and by spatial features of MD and MT components of the magnetic source. The last define two basically different types of possible topology of the magnetic field. The analysis of the structure of M_G in cylindrical coordinates reveals a quasi-cylindrical character of the field structures in the streamer. Traditional for magnetosphere physics, a spherical harmonic representation of the fields appear to be not effective here due to essentially a different decrease of the field along and across the streamer.

The basic features of the behavior of the real characteristic function M_G, can be studied using ideal modelling cases. One of these cases corresponds to a situation when weak dependence of M_G on the coordinate X is ignored. As a test here, we take a simple 1D function $M_G(r_\perp)$, which monotonically decreases with $r_\perp$ by a power-law (no modulations). In accordance with Eqs (17,18) in this case we obtain a *rough tail/streamer structure*, which is described by two cylindric harmonics. These harmonics are: 1) a two-wire-current which can be called a cylindrical 2D dipole and, 2) a solenoidal harmonic which is connected with a Θ current configuration formed by two solenoids. The last may be considered as a kind of a 2D cylindrical toroid (Fig.4). In connection with these two types of harmonics, we distinguish two basic kinds of streamer topology above a solar active region; "dipole tail/streamer" and "toroidal tail/streamer".

In the case of a real characteristic function $M_G(X, r_\perp)$ (16) analogous 3D harmonics are formed, but modified by a presence of *fine structure elements* modulating the *rough tail/streamer*. These are schematically shown in Figs. 5 and 6.

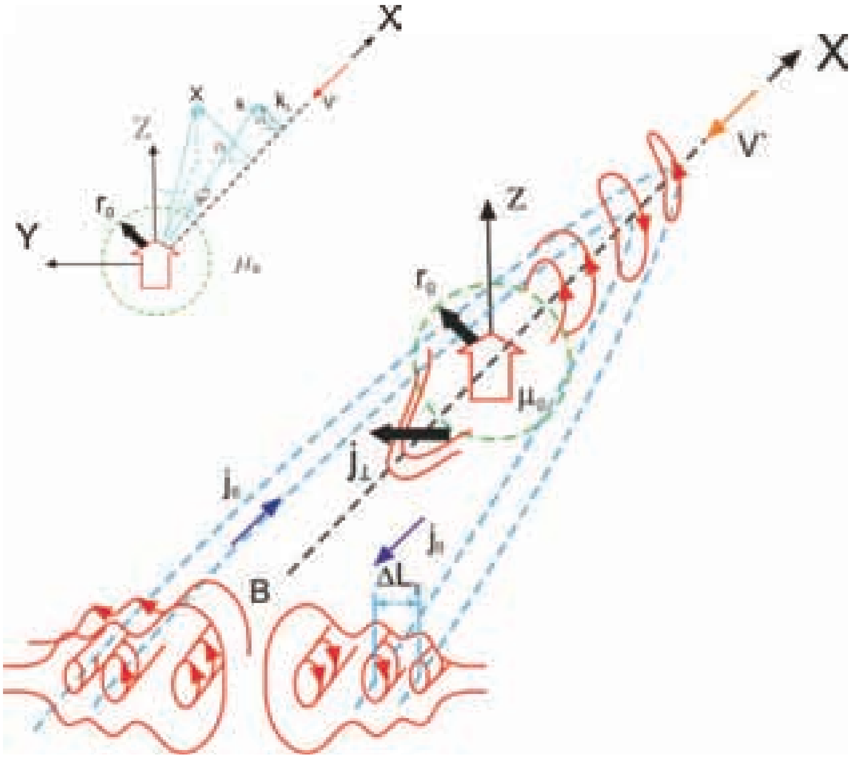

Figure 5: 3D Two-wire-current (2D dipole) system (blue lines). It is defined by the first order derivatives of the characteristic function M, decaying relatively slow with distance X from the source. In the far region of the streamer, fine structure magnetic ropes (red lines) have the scale $\Delta L_y(r_G, X)$. This harmonic is excited by the MD component in the source located on the Sun surface.

The dipole tail/streamer ($\Gamma_{\tau\mu} << 1$) has a rough two magnetic rope configuration, structured by fine magnetic flux rope elements which have a characteristic scale $\Delta L = \Delta L_y$. They appear due to spatial modulation of the characteristic function across the X axis. This type of streamer has "normal component to the neutral sheet plane $X - Y$", $\mathbf{B} = B_n \mathbf{z}_0$. This component slowly decreases with distance X. The two wire-current harmonic is excited mainly by the j_x component of the source current $\mathbf{j}_t$ (Fig. 3).

The toroidal tail/streamer ($\Gamma_{\tau\mu} >> 1$) has two opposite solenoid-type currents which form a "neutral sheet" in the plane $X - Y$ and Θ-type current configuration. The density of the current in the neutral sheet slowly decreases in X direction (i.e., along the tail). The spatial modulation $\Delta L = \Delta L_z$ of the rough structure forms "fine" current sheet elements. This harmonic is generated mainly by the j_y component of the source current (Fig. 4).

According to observations, Magnetic tail and Coronal streamers show a rich collection of topological elements associated with magnetic structures. We can distinguish three basic elements in these visible structures: current sheets, magnetic islands and magnetic ropes. In our resistive model we find current sheets and magnetic ropes which can co-exist. They are produced by interaction of the plasma flow with magnetic sources and may be considered as building blocks of the real 3D streamer. The resulting topology strongly depends on the value of the basic parameter $\Gamma_{\tau\mu}$. In the case of isotropic plasma, the scales ΔL_z and ΔL_y are the same. Magnetic islands are not generated in the case of

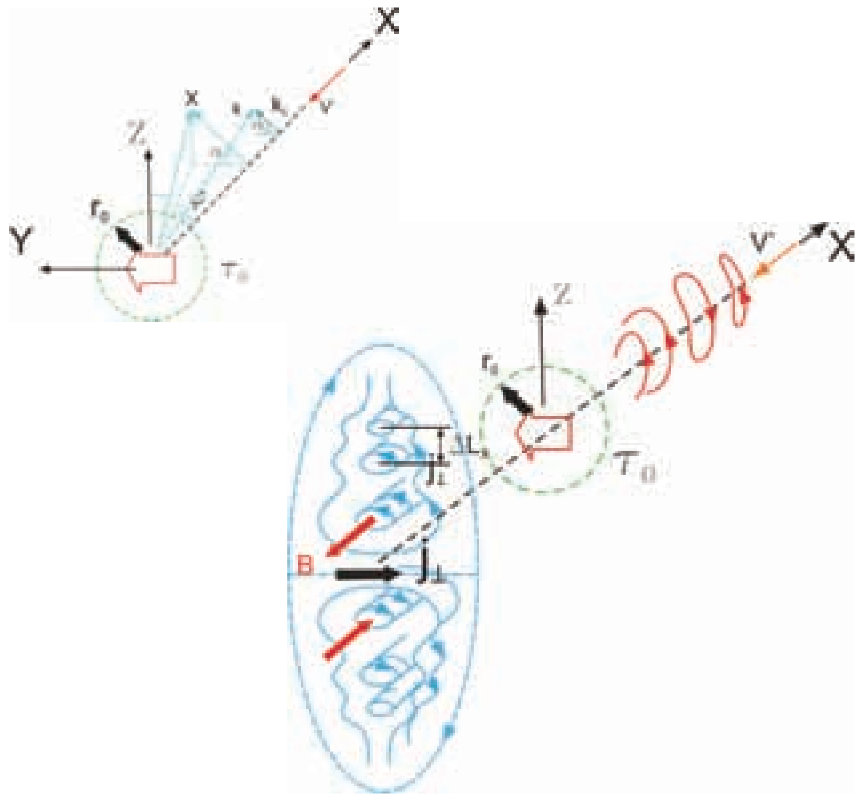

Figure 6: 3D Θ-type current configuration (toroid). It is defined by the second order derivatives of the characteristic function M and decays relatively fast with a distance from the source. Fine structure is formed by a set of current sheets separated at the scale $\Delta L_z(r_G, x)$. Magnetic field (red line) is directed along the flow has reversal on the plane $Z = 0$. This harmonic is excited by the MT component in the source.

M_G considered here, but they may be formed in more complicated situations.

6 Conclusions

Solar streamer 3D structure elements were considered analytically within a plasma kinetic approach. These structure elements were formed by the superposition of a Θ-type current configuration and a two-wire-current configuration which have a spatial modulation resulting in the appearance of ropes and sheets self-consistent with the plasma flow. These fine structure elements are morphologically similar to the observed structures in space plasma. The 3D structures which we study are fully dissipative in the case of a Maxwellian plasma flow. To obtain 3D diamagnetic structures, one needs to operate with plasmas having an anisotropic distribution function.

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