MECHANISM OF RELATIVISTIC JET FORMATION AND GENERATION OF SYNCHROTRON RADIATION

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Abstract

In the MHD description of plasma phenomena the concept of magnetic field lines frozen into the plasma turns out to be very useful. The classical example is the well-known Dungey model in which the time evolution of a flux tube gives a clear physical description of solar wind-magnetosphere interaction. We applied this idea to the plasma accretion on a rotating black hole. First, we present a method of introducing Lagrangian coordinates into the relativistic MHD equations in general relativity, which enables a convenient mathematical formulation for the behaviour of flux tubes. With the introduction of these lagrangian, so–called "frozen–in" coordinates the relativistic MHD equations reduce to a set of nonlinear 1D string equations, and the plasma may therefore be regarded as a gas of nonlinear strings corresponding to flux tubes. If such a tube/string happens to fall into a Kerr black hole, then the leading portion loses angular momentum and energy as the string brakes, and to compensate for these losses, momentum and energy have to be radiated to infinity to conserve energy and momentum for the tube. Inside the ergosphere, the energy of the leading part can be negative, and the rest of the tube then extracts energy from the hole in the form of a torsional Alfvén wave to produce relativistic jet. Numerical calculations show that energy is really extracted from the ergosphere in the form of spiral waves. As a result, a relativistic jet forms, which extends for 30 r_h (event horizon) with a Lorentz-factor ~ 2.5. As the magnetic field has a helical structure, we can expect the existence of synchrotron and curvature radiation from the jet. Using qualitative analysis, we evaluate the possible influence of the reconnection process on the relativistic jet behaviour.

1 Introduction

Jets are widespread in the Universe and reveal themselves in the vicinity of different objects: young stars, active galactic nuclei etc [Novikov, 1990]. In the last few decades

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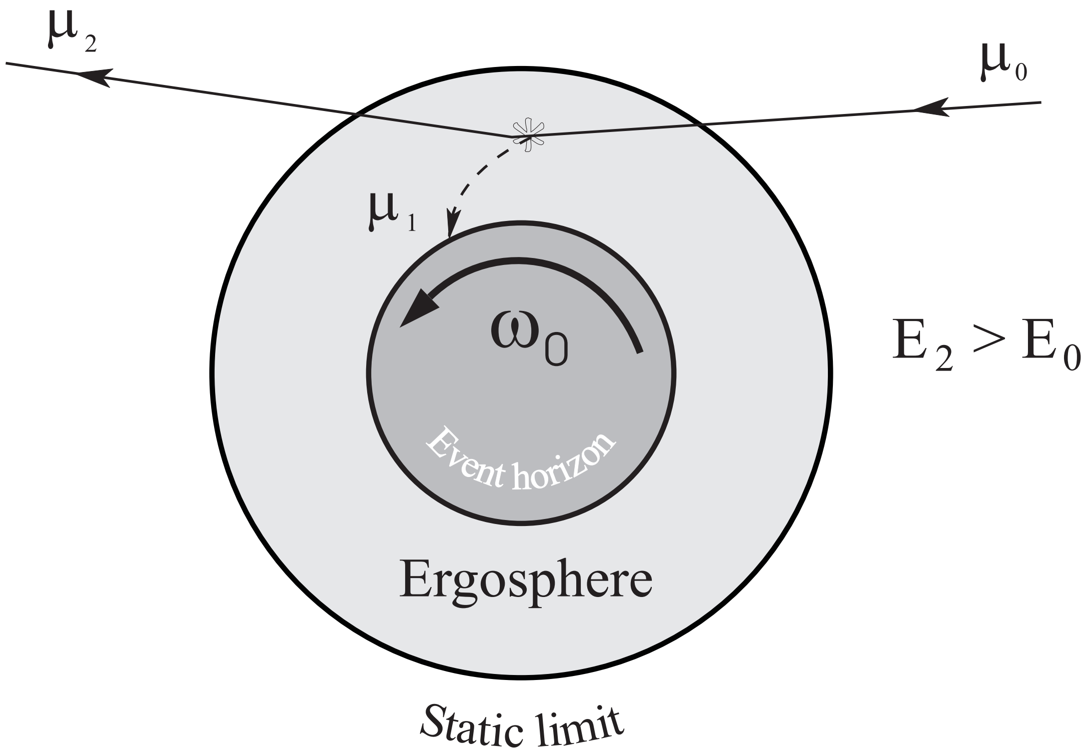

Figure 1: A body falling from a certain distance enters the ergosphere of rotating black hole and explodes at any point close to the black hole surface into two fragments. One fragment is absorbed by the black hole. The other one is ejected from the ergosphere having energy greater than the energy of the falling body.

a lot of mechanisms with jet activity explanation were suggested, however, the nature of this phenomena up to now is cause of debates. Nevertheless, the general consensus is that a key role for such phenomena plays the magnetic field which is frozen in an accreting plasma.

Here we present a new mechanism which is based on the original Penrose process [Penrose, 1969], but instead of particle we consider the interaction of a magnetic flux tube with a spinning black hole. Using Lagrangian coordinates, magnetized plasma can be modelled as a set of flux tubes. This suggestion is bounded with the notion that a plasma embedded in a magnetic field can be represented as a gas of non-linear strings (flux tubes), rather than a gas of particles [Semenov and Erkaev, 1992]. Such an approach, based on examining the behaviour of a test flux tube, can provide a clear physical meaning to the otherwise difficult problems of analyzing the dynamics of space plasmas (see, for example, [Semenov and Erkaev, 1992; Dungey, 1961; Zwan and Wolf, 1976].

2 Basic equations

A convenient mathematical formulation for studying the behaviour of flux tubes can be obtained through the introduction of Lagrangian coordinates into the relativistic magnetohydrodynamic equations (RMHD) of general relativity. The RMHD equations are [Lichnerowicz, 1967];

$$\nabla_i \rho u^i = 0, \tag{1}$$
$$\nabla_i T^{ik} = 0, \tag{2}$$
$$\nabla_i (h^i u^k - h^k u^i) = 0. \tag{3}$$

Here, equation (1) is the continuity equation, equation (2) is the energy-momentum equation, and equation (3) is Maxwell's equation; u^i is the time-like vector of the 4-velocity, $u^i u_i = 1$, and

$$h^i = *F^{ik} u_k \tag{4}$$

is the space-like 4-vector of the magnetic field, $h^i h_i < 0$, $*F^{ik}$ is the dual tensor of the electromagnetic field, and T^{ik} is the stress-energy tensor;

$$T^{ik} = Qu^i u^k - Pg^{ik} - \tfrac{1}{4\pi} h^i h^k, \tag{5}$$

where

$$P \equiv p - \frac{1}{8\pi} h^k h_k, \;\; Q \equiv p + \varepsilon - \frac{1}{4\pi} h^k h_k. \tag{6}$$

Here, p is the plasma pressure, P is the total (plasma plus magnetic) pressure, ε is the internal energy including ρc^2, and g_{ik} is the metric tensor with signature $(1, -1, -1, -1)$. Generally speaking, $\nabla_i h^i = 0$, but we can find a function q such that $\nabla_i q h^i \neq 0$. Then, using (1), the Maxwell equation (3) can be rewritten in the form of a Lie derivative;

$$\frac{h^i}{\rho} \nabla_i \frac{u^k}{q} = \frac{u^i}{q} \nabla_i \frac{h^k}{\rho}, \tag{7}$$

and we can therefore introduce coordinates τ, α such that;

$$x^i_\tau \equiv \frac{\partial x^i}{\partial \tau} = \frac{u^i}{q}, \;\; x^i_\alpha \equiv \frac{\partial x^i}{\partial \alpha} = \frac{h^i}{\rho}, \tag{8}$$

with new coordinate vectors $u^i/q, h^i/\rho$ tracing the trajectory of a fluid element and the magnetic field in a flux tube. Using (8), the energy-momentum equation (3) can be rearranged to form a set of string equations in terms of the frozen-in coordinates,

$$-\frac{\partial}{\partial \tau}\left(\frac{Qq}{\rho} x^l_\tau\right) - \frac{Qq}{\rho} \Gamma^l_{ik} x^i_\tau x^k_\tau + \frac{\partial}{\partial \alpha}\left(\frac{\rho}{4\pi q} x^l_\alpha\right) + \frac{\rho}{4\pi q} \Gamma^l_{ik} x^i_\alpha x^k_\alpha = -\frac{g^{il}}{\rho q} \frac{\partial P}{\partial x^i}, \tag{9}$$

where $q = 1/(g_{ik} x^i_\tau x^k_\tau)^{1/2}$ and Γ^l_{ik} is the Christoffel symbol.

3 Nonlinear string in Kerr metric

To investigate the interaction of the flux tube with a black hole we use numerical calculation, however, the preliminary qualitatively estimation of such interaction is very useful.

The Kerr metric in Boyer-Lindquist coordinates is given by the following line element [Misner at al. 1973];

$$
\begin{aligned}
ds^2 &= (1-\frac{2Mr}{\Sigma})dt^2 - \frac{\Sigma}{\Delta}dr^2 - \Sigma d\theta^2 - \\
&-(r^2+a^2+\frac{2Mra^2}{\Sigma}\sin^2\theta)\sin^2\theta\, d\varphi^2 + \frac{4Mra}{\Sigma}\sin^2\theta\, d\varphi\, dt,
\end{aligned}
\tag{10}
$$

where

$$
\Delta = r^2 - 2Mr + a^2,\ \ \Sigma = r^2 + a^2\cos^2\theta. \tag{11}
$$

Here M and a are the mass and angular momentum of the hole, respectively, and we use a system of units in which $c = 1, \quad G = 1$.

Using the variation method for the string, we can get conservation laws. For cyclic variables τ and ϕ, the energy and angular momentum conservation laws for the flux tube can be written as,

$$
\int_{\alpha_1}^{\alpha_2} \frac{Q}{w\rho}(g_{tt}t_\tau + g_{t\varphi}\varphi_\tau)d\alpha = E, \tag{12}
$$

$$
\int_{\alpha_1}^{\alpha_2} \frac{Q}{w\rho}(g_{t\varphi}t_\tau + g_{\varphi\varphi}\varphi_\tau)d\alpha = -L, \tag{13}
$$

if there is no flux of energy and angular momentum through the ends α_1, α_2 of the flux tube.

Bearing in mind these conservation laws and different rotation of the space around the black hole, we inevitably are lead to the generalization of the Penrose mechanism for the string case.

Due to an inhomogeneous rotation, the string begins to be stretched and twisted (Fig. 2), that entails the magnetic field amplification and as a result string braking. The leading part of the flux tube intensively losses angular moment and energy and parts of the tube with

$$
\Omega_{tube} = \frac{\varphi_\tau}{t_\tau} < \omega_0 = -\frac{g_{t\varphi}}{g_{\varphi\varphi}} \tag{14}
$$

must have negative momentum with respect to the rotation of the hole (13). In contrary, the rest parts of the sting must gain the positive momentum to compensate the losses for the string as a whole (13). Taking into account the fact that the stretching and twisting of the falling flux tube is most pronounced close to and especially inside, the ergosphere where $g_{tt} < 0$ and the energy of a particle or tube element can be negative [Misner at al. 1973]. This means that other parts of the flux tube gain the abundance of positive energy to support (12). Redistributing this energy along the string may lead to its extraction from the ergosphere with energy greater then the initial energy of the string, E [Semenov at al., 2002]. To some extent, this is similar to the Penrose [1969] process (fig. 1), but now we do not need to invoke the interaction or decay of particles or tubes, since just a single tube can extract energy from the hole.

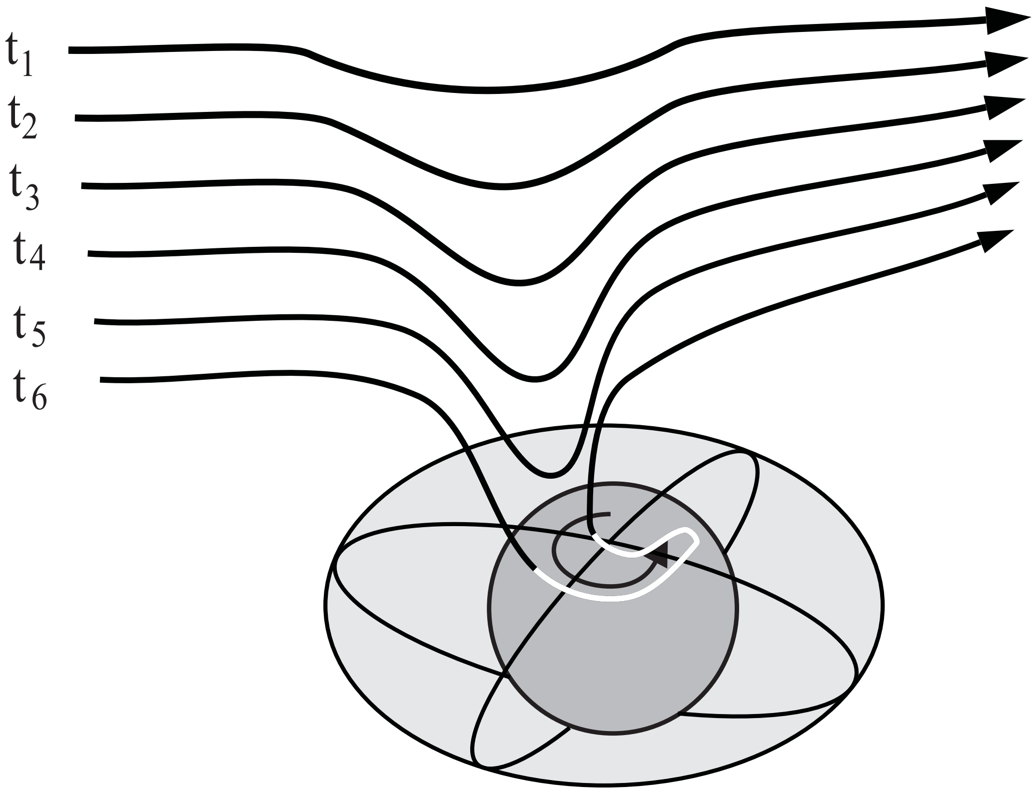

Figure 2: Stretching of a magnetic flux tube. The deeper the magnetic tube falls in the hole, the more the tube is stretched, the stronger the magnetic field gets, the slower the tube rotates, the more negative momentum is generated, and the more positive momentum escapes to infinity.

Basing on a estimation of the flux tube behaviour in the Kerr metric, the continuous process of energy extraction is the following: The deeper the tube falls in the hole, the more the tube is stretched, the stronger the magnetic field gets the slower the central part of the tube rotates, the more negative momentum and energy are generated, and the more positive momentum and energy are created. Eventually, plasma is ejected from the ergosphere producing relativistic jet [Semenov at al. 2004]. However, to confirm our assumptions and to investigate the suggested mechanism in depth, we performed a set of numerical simulations.

4 Results of the numerical calculation

To investigate the string evolution in spinning Kerr geometry we use numerical calculation, which is based on the TVD method [Semenov at al., 2002, 2004]. This is a well-known method which is widely applicable for solving hyperbolic equations [Toth and Odstrcil, 1996; Toth, 1998].

The numerical calculation completely confirms our assumptions. It turned out that the physics of the energy extraction from a rotating black hole and a relativistic jet formation are rather simple. In the spinning Kerr geometry, the leading part of the falling flux tube

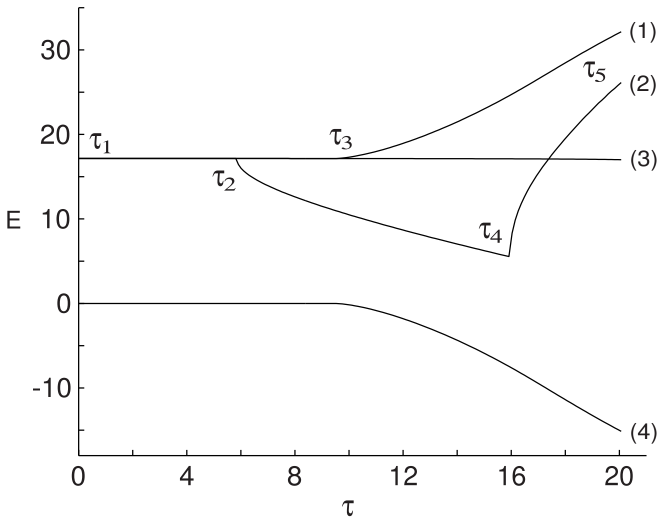

Figure 3: Energy string behaviour during the calculation. Line (1) is the total positive energy of the string, (2) is the positive energy of the string outside the ergosphere, (3) is the total energy of the string (it is conserved during simulation) and (4) is negative energy of the string. Different string times ($\tau_1 \ldots \tau_5$) mark different stages of the string behaviour: τ_1 is the start time of simulation, τ_2 is the central part of the string crosses the static limit, τ_3 is the beginning of negative energy creation, τ_4 is the initial time of jet creation, τ_5 is the end point of simulation.

progressively loses angular momentum and energy (Fig. 3) as the string/tube brakes, which leads to the creation of negative energy inside the ergosphere (Fig. 3, 4b). To compensate the losses of energy and angular momentum for the tube as a whole, the positive energy and angular momentum have to be generated for the trailing part of the tube (Fig. 3). As it was pointed out, this is a nonlocal variant of the Penrose process.

For the first "confinement" stage it turns out that both parts with negative and most of the positive energy are localized in the narrow layer near the event horizon (Fig. 4b) [Semenov at al., 2002]. The created positive energy and momentum can not leave the ergosphere since they are closed there by plasma accretion. However, the string stretching is a continuous process that leads to a magnetic field amplification and as a result to an increase of MHD waves velocities. An energy accumulation process can not last for infinitely and after some time parts of the string with abundance of positive energy and momentum are extracted from the ergosphere in the form of torsional Alfvénic waves producing relativistic jets (Fig. 3,4c) [Semenov at al., 2002, 2004].

Propagating with relativistic speed away from black hole, the jet carries out the energy and angular momentum to the ambient medium (fig. 4c). In our simulation we sorted out different meaning of the rotational parameter a, however, the best result we get using an extremely rotating black hole with $a = 0.9999$, using this parameter, we observe the jet structure creation with a Lorenz-factor ~ 2.5 (Fig. 4c). Considering the further jet behaviour, we inevitably come up to a situation where a region appears with strong magnetic field and low density. This is the result of string fixation in the ergosphere by means of negative energy gained on the one hand and jet development on the other

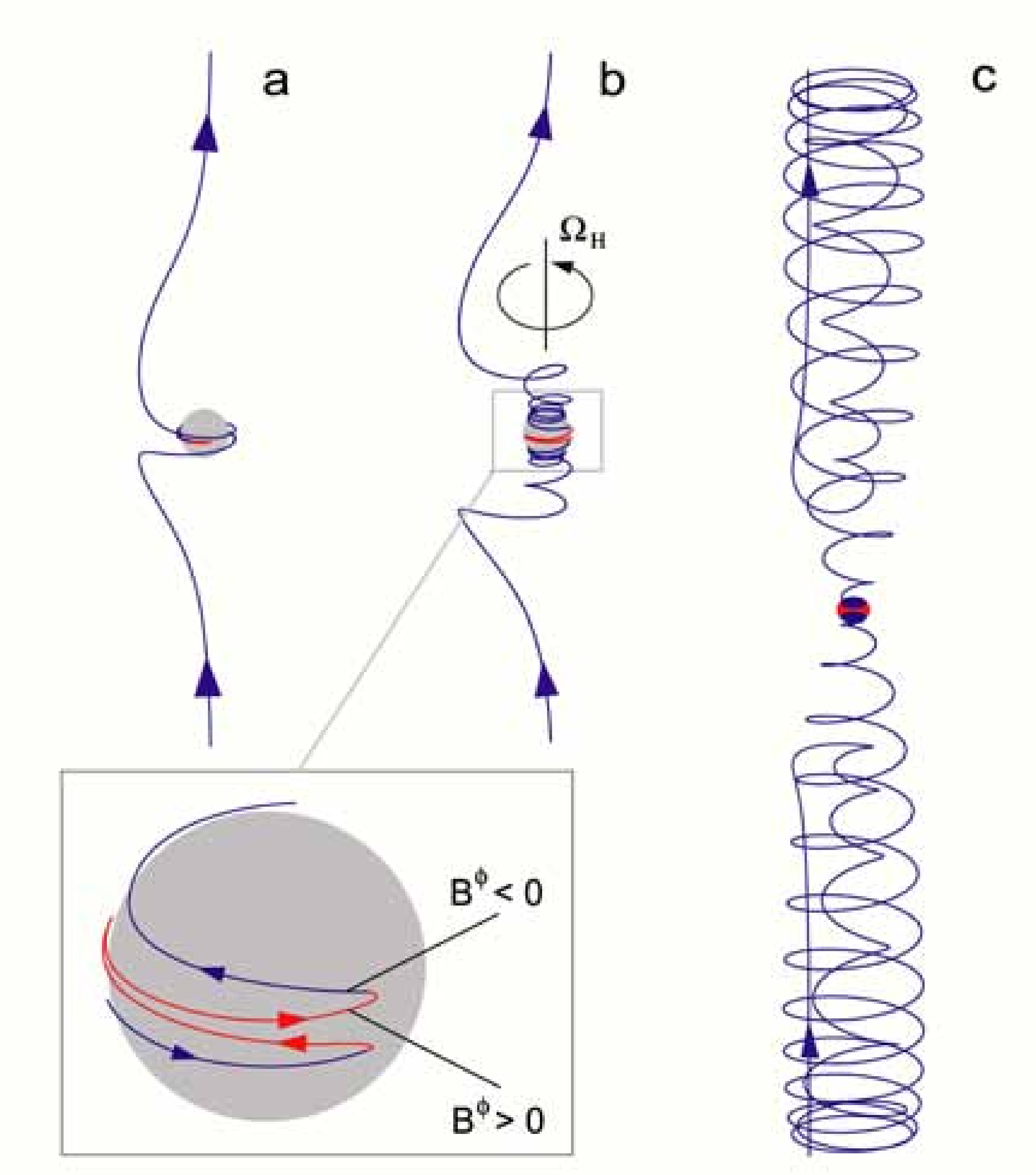

Figure 4: Different moments of simulations: (a) – corresponds to the negative energy creation onset (part of the string with negative energy labelled by red color), (b) – beginning of the spiral structure creation, (c) – the last moment of simulation, (d) – magnetic flux tube structure near the event horizon.

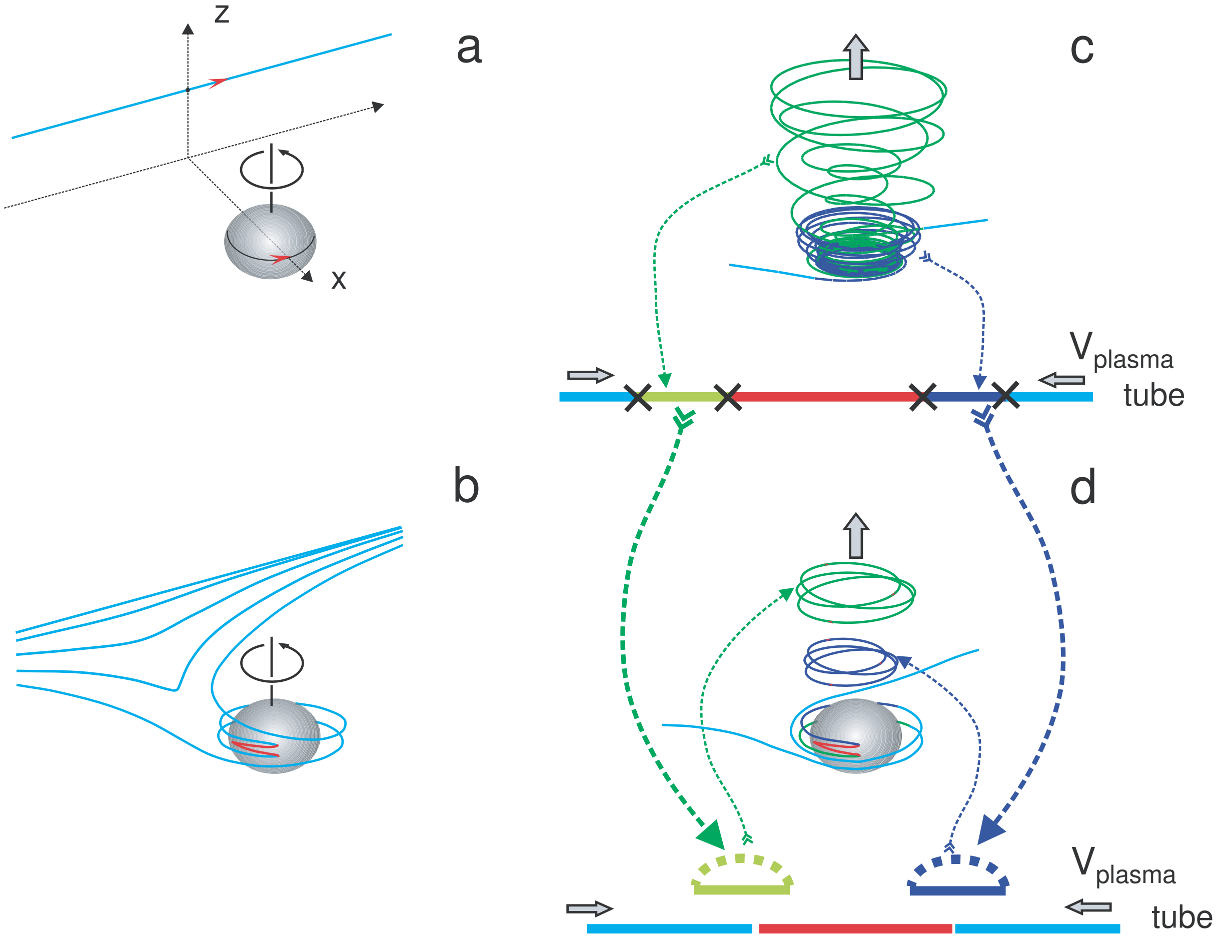

Figure 5: Reconnection influence on the relativistic jet behaviour. Involving in the differential rotation around the event horizon, magnetic flux tube (a) begins stretched and twisted (b), that leads to energy extraction which is attended by the relativistic jet creation (c). The reconnection process may lead to the creation of the independent, closure string objects (labelled by green and blue colours) which move in a direction away from the black hole (d).

hand. Obviously, to release Maxwellian tensions, the next step of our calculation has to be taking into account the reconnection process.

5 The role of reconnection: qualitative evaluation

Bearing in mind the changes which follow for reconnection, we can draw a short qualitative analysis of the further jet structure development. The results of a such analysis are shown in Fig. 5, where an initial string configuration is oriented normal to the black hole axis rotation. Due to the interaction of the string and the black hole, the relativistic jet is formed as mentioned above (Fig. 5a,b). The process of relativistic jet creation is accompanied by a complication of the flux tube structure (Fig. 5c), that may lead to magnetic field reconnection. The reconnection process, in turn, changing the string topology, may entail the separation of parts of the flux tube from the main jet structure. Some string portions turn out to be disconnected from other parts and, most significantly they may be isolated from the flux tube which is bounded with the black hole (Fig. 5d). This means that such independent and closure parts of the flux tube may leave a black hole vicinity carrying over the energy and momentum (Fig. 5d), that eventually leads to an impulse description of the energy extraction process as a whole.

6 Synchrotron and curvature radiation

Due to the differential rotation of the string, the magnetic field can increase a lot that must lead to a burst of synchrotron radiation in the vicinity of a black hole as [Beskin, 2005],

$$\tau_s \approx \frac{1}{\omega_B}\left(\frac{c}{\omega_B r_e}\right) \sim 10^{-15}\text{s}. \tag{15}$$

Here τ_s is the synchrotron highlighting time, $\omega_B = eB/m_e c$ is the Larmor frequency, $r_e = e^2/m_e c^2$ is the electron radius. Therefore, the charged particles very fast loose energy and move along the magnetic lines generating the curvature radiation with maximum frequency [Zheleznyakov, 1977].

$$\omega_{cur} = 0.29\frac{c}{R_c}\gamma^3. \tag{16}$$

Here R_c – radius of curvature and γ - Lorentz factor.

Relying on our qualitative analysis we may expect the appearance of a closure string structure which carries away energy and momentum (Fig. 5d). Apparently, such plasmoids may propagate for huge distances from the central source as low-luminosity objects losing the energy only in the form of curvature radiation.

Supplement

Movies of the jet calculations are available from `http://geo.phys.spbu.ru/~ego/`

Acknowledgments

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