RF–WAVE PROPAGATION IN THE ANISOTROPIC SPACE PLASMA

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Abstract

A method of RF wave propagation analysis is presented. This method provides transparent formulae for the wave refractive index in a hot and anisotropic plasma. They are simple enough to be discussed in analytical form. The modification of the whistler and ordinary waves propagation due to plasma finite temperature and anisotropy is shortly discussed. The extraordinary mode propagation is used as an example to present this method in detail. The extraordinary wave energy focusing in the direction perpendicular to the magnetic field is considered for different plasma parameters. This mode energy trapping in the vicinity of the equatorial plane of magnetosphere is considered with a simple model plasma distribution function. It is emphasized that this kind of trapping provides convincing explanation for Auroral Kilometric Radiation (AKR) observations at frequencies close to the second harmonic of the electron gyrofrequency.

1 Introduction

While describing the Radio Frequency wave propagation in the space plasma it often appears necessary to construct a theory that could be transparent enough for an analytical analysis, but at the same time, could take into account the observable properties of the magnetospheric plasma and particularly its finite temperature and anisotropy. Moreover, the analysis of propagation in a thermal plasma provides validity conditions of the simplest approximation, i.e. cold plasma theory.

The realistic plasma model for numerical wave simulations also requires such a simple theory, which may provide an efficient computer code.

In several of our previous publications (see Sazhin 1981, Sazhin 1982b, Kobelev and Sazhin 1983, 1984, Sazhin and Sazhina 1982, Majewski and Sazhin 1984) we attempted to construct such a theory basing on the earlier results of Sazhin et al. (1981), who

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assumed that the influence of protons on the wave propagation would be small and the electron distribution function was taken in the form

$$f(v_\perp, v_\parallel) = \left(j\pi^{3/2} w_\perp^{2j+2} w_\parallel\right)^{-1} v_\perp^{2j} exp\left(-\frac{v_\parallel^2}{w_\parallel^2} - \frac{v_\perp^2}{w_\perp^2}\right) \tag{1}$$

where $w_parallel$ and w_perp are the root mean square electron velocities parallel and perpendicular to the magnetic field, respectively. The parameter $A_e = (j+l)\frac{w_\perp^2}{w_\parallel^2} = 1,\ 2,\ \ldots$ is proportional to the degree of anisotropy. The values of $w_\parallel$ and $w_\perp$ were assumed to be so small that the following inequalities were valid

$$\left(\frac{1}{2}\right)\frac{k_\perp^2 w_\perp^2}{\Omega^2} \ll 1 \tag{2}$$

$$\frac{k_\parallel^2 w_\parallel^2}{(\omega - n\Omega)^2} \ll 1 \tag{3}$$

where $k_\parallel$, $k_\perp$ – the components of the wave vector in the direction parallel and perpendicular to the external magnetic field, respectively; ω, Π and Ω are wave frequency, electron plasma frequency and gyrofrequency, respectively; and $n = O$, 1 and 2.

Further, it was assumed that the square of the wave refractive index N^2 differs only slightly from the corresponding wave refractive index in a cold plasma N_0^2

$$|N^2 - N_0^2| \ll N_0^2 \tag{4}$$

Plasma was considered to be homogeneous in a sense that the wave dispersion characteristics at each point did not depend on the plasma inhomogeneity at this point.

However, the paper of Sazhin et al. (1981) contained an algebraic mistake: A_e in the expression for $E_{13}^{(2)}$ is to be replaced by $2A_e - 1$ and in the expression for $E_{23}^{(2)}$ by $2 - A_e$; the general formulae (6) - (14) in this paper are correct. This mistake did not influence the results of the subsequent papers when $A_e = 1$, or $\Theta = O$, $\pi/2$ for arbitrary A_e. In most cases its influence is negligible even when $A_e = 1$, so that the conclusions of these papers remain valid, although the numerical values of the coefficients in some formulae are to be changed. This problem will be discussed in detail in our future publications. Here we restrict ourselves to summarizing some results of the previous papers, which are not influenced by this mistake.

First, the wave energy is focused along the magnetic field for $\omega = 0.55\Omega$ when $A_e = 1$. This result finds an experimental confirmation in the observed maximum value ill the chorus spectrum (Sazhin, 1982b).

Second, the wave may be trapped in a density as well as temperature duct. Trapping in a density enhancement duct takes place tor $\omega < 0.55\Omega$ when $A_e = 1$. Trapping in a density depletion duct is possible only for $A_e = 1$ and $\omega > 0.55\Omega$. A temperature enhancement duct may trap a whistler wave if $\omega < 0.55\Omega$ for $A_e = 1$. A temperature depletion duct is able to trap a whistler if $A_e = 1$ and $\omega > 0.55\Omega$.

Now, we proceed to the main subject of this paper i.e. to analysis of the extraordinary wave propagation. The generalization for the extraordinary mode was proposed by Majewski and Sazhin (1984). This mode propagation has so far only been considered when using numerical methods (Venugopal and Viswanathan, 1982). The necessity to provide a transparent model for the extraordinary wave is primarily connected to the fact that AKR and other planetary emissions propagate through the plasma in this mode (Majewski and Popielawska, 1980; Grabbe, 1981; Shawhan and Gurnett, 1982; Gurnett et al., 1983; Anderson et al., 1983; Pritchett, 1983).

In the following part of discussion we will proceed along the lines of the paper by Majewski and Sazhin (1984). First cold plasma approximation will be described, then a general solution for thermal and anisotropic plasma with the distribution function (1). From this solution the results concerning wave energy focussing an trapping will be derived.

2 Extraordinary Mode Propagation in a Cold Plasma

We restrict our analysis to almost perpendicular propagation of the wave. This assumption considerably simplifies the analysis, while the most significant features of the wave propagation are still retained. The corresponding analytical solution for the propagation at finite angles to the magnetic field appears to be so complicated that its merit, when compared with the numerical results, remains doubtful.

A formula for extraordinary mode refractive index in the case of propagation almost perpendicular to the magnetic field in a cold plasma can be obtained from the Appleton–Hartree dispersion equation

$$N_0^2 = N_{0\perp}^2 \left(1 + a_0 \Phi^2\right) \tag{5}$$

where

$$N_{0\perp}^2 = \frac{(1-X)^2 - Y^2}{1 - X - Y^2} \tag{6}$$

$$a_0 = \frac{X(X-1)}{1 - X - Y^2} \tag{7}$$

and $X = \dfrac{\Pi^2}{\omega^2}$, $Y = \dfrac{\Omega}{\omega}$, $\Phi = \dfrac{\pi}{2} - \Theta$, $|\Theta| \ll 1$. In deriving (5) proton contribution was neglected.

It follows from (6) that $N_{0\perp} = O$ when $Y = \pm(1 - X) \equiv Y_{r,l}$. Introducing the notation Y_r and Y_l , we recall Stix's (1961) abbreviations

$$R = \frac{1-X}{(1-Y)} \tag{8}$$

$$L = \frac{1-X}{(1+Y)} \tag{9}$$

From (8)-(9), it is evident that $R = O$ when $Y = Y_r$ and $L = O$ when $Y = Y_l$. From (6) it may also be seen, that $N_{o\perp} \to \infty$ when Y approaches

$$Y_{uh} = (1-X)^{1/2} \tag{10}$$

Equation (10) determines the upper hybrid resonance frequency.

The perpendicular extraordinary wave, with dispersion equation in the form (6), can exist only if the right hand side of (6) is positive, which is possible when

$$Y_{uh} < Y \ < \ Y_l \tag{11}$$

$$Y \ < \ Y_r \tag{12}$$

Formula (5) was analyzed in detail by Majewski and Sazhin (1984) with the values of $\lambda = \frac{X^{1/2}}{Y} = 0.5,\ 1,\ 2$. The values of λ considered here are characteristic for the magnetospheric plasma in the region outside the plasmasphere (Akasofu and Chappell, 1973; Curtis, 1978). They will be used in our further analysis.

When the direction of wave propagation diverges from the perpendicular, the dependence of N_0 on Φ is controlled by the parameter a_0. Therefore Majewski and Sazhin (1984) proposed to divide the wave propagation pattern into six classes. The behavior of the function $N_\parallel(N_\perp)$ was proposed as the criterion of this division; where $N_\parallel$ and $N_\perp$ are the components of the refractive index in the direction parallel and perpendicular to the rnagnetic field, respectively. Those characteristic types are: type "*a*" corresponding to $a_0 \to -\infty$, type "*b*" $a_0 \to \infty$, "*c*" to $a_0 \ll 0$, type "*d*"to $a_0 \gg 0$, type "*e*" to $a_0 = 0$, and type "*c*" to $a_0 = 1$. Taking into account the fact that the direction of group velocity, and consequently the direction of energy propagation, is perpendicular to the curve $N_\parallel(N_\perp)$ (Stix, 1961), we may conclude that the different types of these curves correspond to the different types of wave energy propagation. In particular, in cases "*a*" and "*b*" the energy propagates almost along magnetic field even when the wave normal is nearly perpendicular to it. For type "*e*" the direction of energy propagation coincides with the wave normal direction. For type "*f*" the wave energy is focused in a direction perpendicular to the magnetic field. This last case is particularly important for the study of wave propagation in the space plasma and will be considered later.

As follows from (7), types "*a*" and "*b*" wave energy propagation are realized when Y diverges or approaches Y_{uh} with λ being fixed, type "*e*" is realized when $X = 1$ or $X \to 0$, which corresponds to $Y = \lambda^{-1}$ or $Y \to 0$ when λ is fixed, and type "*f*" is realized when $Y \cong 1.0$, 0.83 and 0.48 for $\lambda = 0.5$, 1.0 and 2.0, respectively. We can expect that the most favorable conditions for wave energy transport across the magnetic field, and in particular Auroral Kilometric Radiation energy transport from the inner magnetosphere towards the magnetospheric tail, occur for these frequencies when the type "*f*" is realized, and less favorable condition for the types "*a*" and "*b*". This conclusion should be considered as preliminary because in the analysis conducted in this section we did not take into account finite electron temperature, anisotropy as well as the magnetospherie inhomogeneity. This will be done in the following section.

3 Generalization of (5) on Propagation in a Hot Anisotropic Plasma

For extraordinary mode propagation in a hot anisotropic plasma wtith a distribution function in the form (1), formlula (5) was generalized by Majewski and Sazhin (1984) using the same technique as in Sazhin and Sazhina (1982), and Sazhin (1982a) for analysis of whistler and ordinary mode propagation when (2)–(4) are valid. After inserting (5) into formula (15) of Sazhin and Sazhina (1982) we obtained an explicit formula for extraordinary mode refractive index in a hot anisotropic plasma with the distribution function in the form (1)

$$N^2 = N_{0\perp}^2(1 + a_\perp\beta + (a_0 + a_\Phi\beta)\Phi^2) \tag{13}$$

where

$$a_\perp = \frac{A_e Y^2(1 - 2X + X^2 + 7Y^2 - 4XY^2 - 8Y^4)}{(4Y^2 - 1)(1 - X - Y^2)^2} \tag{14}$$

$$a_\Phi = a_{\Phi 0} + A_e a_{\Phi A} \tag{15}$$

and the following formulae are valid

$$A_e = (j+1)\frac{w_\perp^2}{w_\parallel^2}$$

$$\beta = \frac{1}{2}\left(\frac{w_\parallel}{c}\right)^2 \cdot \left(\frac{\Pi}{\Omega}\right)^2$$

$$a_{\Phi 0} = \frac{\Sigma_{i=1}^5 a_i X^{i-1}}{(1 - Y^2)^3 (1 - X - Y^2)^2}$$

$$a_{\Phi A} = \frac{\Sigma_{i=1}^6 b_i X^{i-1}}{(4Y^2 - 1)(1 - Y^2)^2 (1 - X - Y^2)^3}$$

$$
\begin{aligned}
a_1 &= -1 + 8Y^2 - 21Y^4 + 21Y^6 - 6Y^8 - Y^{1O} \\
a_2 &= 4 - 22Y^2 + 38Y^4 - 26Y^6 + 6Y^8 \\
a_3 &= -6 + 22Y^2 - 27Y^4 + 12Y^6 - Y^8 \\
a_4 &= 4 - 1OY^2 + 8Y^4 - 2Y^6 \\
a_5 &= -1 + 2Y^2 - Y^4 \\
b_1 &= -1 + 6Y^2 - 16Y^4 + 18Y^6 - 3Y^8 - 8Y^{1O} + 4Y^{12} \\
b_2 &= 5 - 26Y^2 + 34Y^4 + 1OY^6 - 43Y^8 + 20Y^{10} \\
b_3 &= -10 + 44Y^2 - 40Y^4 - 32Y^6 + 58Y^8 - 20Y^{10} \\
b_4 &= 10 - 36Y^2 + 34Y^4 - 8Y^8 \\
b_5 &= -5 + 14Y^2 - 13Y^4 + 4Y^6 \\
b_6 &= 1 - 2Y^2 + Y^4
\end{aligned}
$$

$N_{0\perp}$ and a_0 are determined in a corresponding manner by the formulae (6) and (7). When presenting (13)-(15) we used the notation which does agree with that used by Budden (1961, 1983).

It should be emphasized that formulae (13)-(15) provide a consistent picture of the propagation in an anisotropic, thermal plasma. Every parameter in those formulae may be easily interpreted as connected to some aspect of propagation, i.e. $a_\perp$ is responsible for the strictly perpendicular propagation, etc. Those formulae provide an efficient code for the computer simulation of the extraordinary waves. One point of the wave–ray trajectory requires only 100 add–and–multiply operations.

Checking formulae (14)–(15) we have compared the values of $a_\perp$ and a_Φ obtained from these formulae with numerical values obtained from the more general formula (15) of Sazhin and Sazhina (1982). Both values coincided to an accuracy of 0.01% for all observable plasma parameters.

Now we are able to demonstrate how much physical information may be derived from this simple model. As one can see from (13)–(14), the influence of finite temperature on strictly perpendicular propagation is most important for $Y \to Y_{uh}$ and $Y \to 1/2$. The first case corresponds to the wave frequency approaching the upper hybrid frequency, where $N_{0\perp} \to \infty$. The importance of the influence of finite temperature on the wave propagation at this frequency could be predicted even from the theory of wave propagation in a cold plasma. At the same time one cannot predict the influence of the thermal corrections to the value of $N_{0\perp}$ when $Y \to 1/2$ while considering only the cold plasma approximation, as at this frequency the value of $N_{0\perp}$ remains finite (see Majewski and Sazhin, (1984)).

Thus the small value of N_0 can not be considered in general as a validity criterion for the cold plasma approximation. A similar property of wave propagation was pointed out earlier in Sazhin's (1982a) analysis of ordinary mode propagation. For strictly perpendicular propagation of this wave, the influence of the finite temperature is most important when $Y \to 1$. For oblique extraordinary mode propagation, thermal corrections to the value of N_0 are most important when $Y \to 1$ and $Y \to 1/2$, as in the case of ordinary mode propagation, and also when $Y \to Y_{uh}$. One can not suppose that the thermal corrections to the value of N_0 are infinite when the wave frequency approaches the above–mentioned frequencies, because in such a case inequality (4), which was used in derivation of (13), would be violated. We can only say that the thermal corrections become important when the wave frequency approaches these frequencies, even when the plasma temperature is small. Thus our theory is not valid in the close vicinity of these frequencies.

Formulae (6)–(7) and (14)–(15) can be simplified in some limiting cases. In particular, we can consider high frequency extraordinary mode propagation when $X \to 0$ and $Y \to 0$. In such a case $a_0 \to 0$, $a_\perp \to 0$, $a_{\Phi 0} \to -1$, $a_{\Phi A} = 1$ and formula (13) reduces to

$$N^2 = 1 + (A_e - 1)\Phi^2 \tag{16}$$

From (16) it follows that the angular dependence of N remains of type "e", for $A_e = 1$ corresponding to isotropic wave propagation. For $A_e = 24$ this dependence is of type "f", which corresponds to wave energy focusing in a direction perpendicular to the magnetic field. We may recall that for ordinary waves the corresponding formula (16) would have the form (Sazhin, 1982a) $N^2 = 1 + (1 - A_e)\Phi^2$.

Let us now consider strictly perpendicular extraordinary mode propagation by setting $\Phi = 0$ in formula (13). The temperature dependence of N^2 is controlled now by the parameter $a_\perp$.

As follows from (14) $a_\perp$ remains finite and negative when $Y \to Y_l$; when $Y \to Y_{uh}$, then $a_\perp \to \infty$ for $\lambda = 0.5$ or 1. From (14) it also follows that $a_\perp \to \pm\infty$ when $Y \to 1/2$ and $a_\perp$ changes its sign in the vicinity of $Y = 1/2$. Finally, it is important to notice that $a_\perp \to 0$ when $Y \to 0^+$ even for finite X, which is consistent with formula (16) because $a_\perp$ is a linear function of A_e.

It follows from (13) that the thermal corrections for strictly perpendicular propagation are proportional to $w_\perp^2$ and do not depend on the value of $w_\parallel$. As was stressed by Sazhin (1982a), the situation for ordinary mode is quite different: thermal correction for perpendicular propagation are proportional to $w_\perp^2$. These properties of ordinary and extraordinary modes can be easily explained from a physical point of view: in the case of perpendicular propagation the electric vector of an ordinary (extraordinary) wave is directed strictly along (perpendicular to) the external magnetic field.

Let us further consider the functions $a_\Phi(Y)$. The behavior of this function appears to be rather complex. For our further analysis it is important to compare the functions $a_\Phi(Y)$ and $a_0(Y)$. One can expect that the influence of the finite electron temperature on the shape of the curves $N(\Phi)$ will be especially important when $a_\Phi \gg a_0$ or when $a_0 \cong 1$. In the first case the term $a_\Phi\beta$ appears to be comparable or larger than a_0, in spite or the fact that for magnetospheric conditions $\beta \ll 1$. In the second case, even a small term $a_\Phi\beta$ could cause changes in the group velocity in a direction perpendicular to the magnetic field. It follows from (7) and (15) that the first case is realized when $Y \cong 1$, $1/2$ or Y_{uh} for different values of X and λ or when $X \cong 1$ or 0 for different values of Y and λ. The second case is realized when $Y \cong 1$ for $\lambda = 0.5$, when $Y \cong 0.83$ for $\lambda = 1$ and when $Y \cong 0.48$ for $\lambda = 2$.

As has already been pointed out, formula (13) is valid only in the case where (2)–(4) are valid. Therefore, one should carefully examine the validity criteria (see Majewski and Sazhin, 1984) before applying (13) to a concrete space plasma situation.

In the next two sections we shall demonstrate, how formula (13) may be applied to the problem of wave energy focusing and its trapping in the equatorial magnetosphere. In order to have some point of reference, we shall use well known example of Auroral Kilometric Radiation propagating through the magnetospheric plasma.

However, it may be easily applied to any other extraordinary wave propagating through the magnetoactive plasma.

4 Extraordinary Mode Energy Focusing

As has been already said in section 2, the wave is focused in a direction perpendicular to the magnetic field for small values of Φ when the dependence $N(\Phi)$ on these Φ is similar to the type "f". This takes place when

$$N\cos(\Phi) = const \tag{17}$$

For extraordinary wave propagation in a cold plasma, (17) is valid when $a_0 = 1$. For propagation of this wave in a hot anisotropic plasma, which was considered in the previous sections, the condition for (17) to be valid can be presented in the form

$$q \equiv a_0 - 1 + (a_\Phi - a_\perp)\beta = 0 \tag{18}$$

Equation (18) determines those values of Y and β for which extraordinary mode energy focusing takes place. It was noted by Majewski and Sazhin (1984) that energy focusing in the direction perpendicular to the magnetic field for $\lambda = 0.5$ or 1 and $\beta = 0.01$ - 0.02 takes place for Y just above 1. When $\lambda = 0.5$ focusing of waves near $Y = 1$ could be predicted from the theory of propagation in a cold plasma ($a_0 \sim 1$). At the same time, for $\lambda = 1$ no such focusing is predicted. From the previous discussion it is obvious that this focusing results from all increase of a_Φ when $Y \to 1$.

It was proved (Majewski and Sazhin, 1984) that focusing can take place when Y is slightly above Y_{uh} for different values of λ and for small values of β. For $\lambda = 1$ and 2, the values of Y for which the wave focusing can take place when $\beta = 0$ correspond to the values of Y for which $a_0 = 1$, which means that this kind of focusing could be predicted from the theory of propagation in a cold plasma.

We should notice that within the scope of our model, it is not possible to predict the interval of Φ for which focusing is effective, as in the case of the ordinary mode which has been considered by Sazhin (1982a).

As was already stressed in section 2, the possibility of wave energy focusing in directions perpendicular to the external magnetic field enables transport of extraordinary wave energy from the inner magnetosphere to the distant regions of the magnetospheric tail. At the same time, in the process of extraordinary wave propagation in real inhomogeneous plasma, the values of Y and λ change, and thus the condition for wave energy focusing, although satisfied at one point, is violated at another one. Thus we can only say that wave energy focusing makes it easier to transport the wave energy, but still we can not yet state whether the wave energy can really be transported. As has already been pointed out by Sazhin (1982a), one can consider wave trapping in the equatorial region of the magnetosphere as a precondition for effective energy transport from one point within the equatorial plane to another, and particularly from the inner magnetosphere to the distant tail. Thus it seems to be important to consider this kind of trapping of the extraordinary wave, keeping in mind the possibility of application of our theory to the problem of AKR propagation through the magnetosphere.

5 Extraordinary Mode Energy Trapping

Let us suppose the magnetospheric magnetic field to be perpendicular to the equatorial plane of the magnetosphere and the magnetosphere itself to be symmetric with respect to the equatorial plane. Then the wave can be trapped in the vicinity of the magnetospheric equator if it reaches a point z defined by

$$N cos(\Phi) = N_\perp{}' = N_\perp(z) \tag{19}$$

Equation (19) should be satisfied after each crossing of the magnetospheric equator; N and Φ refer to the equatorial plane and $N_\perp{}'$ refers to the point of reflection z (Sazhin 1982a). This equation constitutes a condition for wave reflection at the point z and is a precondition for wave trapping in the vicinity of the magnetospheric equator. For a wave propagating with small Φ in a hot anisotropic plasma, for which formula (13) was derived, equation (19) can be simplified and presented in the form

$$\Phi^2 = \frac{N'^2_{0\perp}(1 + a'_\perp \beta') - N^2_{0\perp}(1 + a_\perp \beta)}{q N^2_{0\perp}} \tag{20}$$

where q is determined by (18) and refers to the equatorial plane. The primed parameters refer to the reflection point of the wave. If the solution of (20) exists for real $\Phi(\Phi^2 > 0)$, then the wave could be trapped. The interval of Φ, for which the wave can be trapped, increases when $q \to O$.

Let us consider the formula (20) in some detail in order to obtain conditions under which a solution of (20) exists. We must first choose a model of magnetospheric plasma distribution. As a zero approximation we can consider plasma density to be constant along the magnetic field lines (as assumed by Sazhin (1982a)) and also to be roughly constant in the equatorial plane (this supposition does not contradict the available experimental data concerning plasma density in the extraplasmaspheric region – Chappell (1972)).

Thus, we shall consider the parameter X to be constant at a given frequency ω. The only parameter that can change during the propagation is Ω i.e. Y. Moreover, we shall assume that the change of this parameter in the magnetospheric equator is negligible when compared with its variation along the magnetic field during one oscillation of the trapped wave. This supposition considerably simplifies the analysis, without neglecting the essential features of the phenomenon.

We may write (20) in the form

$$\Phi^2 = \frac{\zeta \Delta Y}{N^2_{0\perp}} \tag{21}$$

where $\Delta Y = Y' - Y > 0$

$$\zeta = \frac{\frac{d}{dY}\left(N^2_{0\perp}(1 + a_\perp \beta)\right)}{q} \tag{22}$$

The value of the derivative d/dY in (22) is determined primarily by the function $N_{0\perp}(Y)$. This value is always negative and wave trapping is possible ($\Phi^2 > O$) only when $q < O$. In a cold plasma the latter inequality is satisfied when $a_0 < 1$ i.e. for almost all values of Y except those close to Y_{uh}.

From (18) it follows that q must be positive when Y is slightly below 1/2, i.e. $\omega > 2\Omega$ (the corresponding increase of $a_\perp$ in the vicinity of $Y = 1/2$ will not change the sign of the derivative in (22). Assuming that Y decreases when the wave is propagating towards the geomagnetic tail and $\beta > 0.001$, we can expect that the wave is trapped when $1/2 < Y < 1$. When the wave reaches the point where $Y \equiv 1/2$, it becomes untrapped and its energy is effectively scattered away from the magnetospheric equator. Hence we can expect that for a given point of the magnetosphere away from the equator, the presence of the waves with frequencies close to 2Ω would be maximal. This is consistent with the observation of AKR near the second harmonic of a local gyrofrequency by Benson and Calvert (1979).

6 References

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