ON DUST KINETIC ALFVÉN WAVES AND STREAMING INSTABILITY IN A LORENTZIAN MAGNETOPLASMA

N. Rubab*, H. K. Biernat*, N. V. Erkaev†, and D. Langmayr‡

Abstract

Dust kinetic Alfvén waves (DKAW) instability with $\kappa-$distributed ions streaming effects have been examined rigorously in a uniform dusty magnetoplasma. A dispersion relation of low-frequency DKAW instability on the dust acoustic velocity branch is obtained in a low-β Lorentzian plasma. It is found that nonthermality is more effective for dust kinetic Alfvén waves in the perpendicular component having finite larmor radius effects. Lorentzian type charging currents are obtained with the aid of Vlasov theory. Effect of different dust parameters on the growth rates of instability are considered. Damping/instability due to dust charge fluctuation is found to be insensitive to the form of the distribution function for DKAW. Possible applications to dusty space plasmas are pointed out.

1 Introduction

Alfvén waves (AW) in a dusty plasma are strongly modified when a proportion of positive/negative charge resides on the dust grain surface. The presence of dust grains in space plasma may play crucial role on the overall dynamics, i.e., breaking down of frozen-in field condition [Kamaya and Nishi, 2000]. It may also play an important role in the dissipation of magnetic flux in the interstellar clouds [Nakano et al., 2002]. In the low frequency regime, the dynamics of Alfvén waves may be influenced due to the presence of dust because of the dynamical importance of dust grains and the circulation of their mass is essentially determined by their kinetic temperature. If the dust particles are accelerated, the energy gained by them from the Alfvén wave can be sufficient to decrease its amplitude, which in turns can reduce the acceleration efficiency [Prudskikh, and Shchekinov, 2008].

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* *Space Research Institute, Austrian Academy of Sciences, Schmiedlstrasse 6, A-8042 Graz, Austria and Institute of Physics, University of Graz, A-8010 Graz, Austria*

† *Institute of Computational Modelling, 660036, Krasnoyarsk, Russia and Siberian Federal University, 660041, Krasnoyarsk, Russia*

‡ *Virtual Vehicle Competence Center (vif), Inffeldgasse 21a, A-8010 Graz, Austria*

Kinetic Alfvén waves (KAWs) are dispersive AWs with a short perpendicular wavelength comparable with the ion gyroradius, and arise when the dispersion relation of ordinary Alfvén waves is modified by the finite Larmor radius effect or simply by the finite electron inertial effect [Hasegawa and Uberoi, 1982]. Several investigations on KAWs including different effects and their coupling with ion acoustic wave (IAW) [Yu and Shukla, 1878; Saleem and Mahmood, 2005] as well as linear and nonlinear dynamics of dust kinetic Alfvén wave (DKAW) coupled with dust acoustic waves has been devoted so far[Yinhua et al., 2000; Rubab et al., 2009], which are driven by the polarization drift of the ion/dust and bending of the magnetic field lines.

Most of the studies mentioned above consider the case when the particle contributions are distributed according to a Maxwellian distribution function. Observational studies of most natural space plasmas dealing with wave–particle interaction are observed to possess a non-Maxwellian high-energy tail component, with the spectral index κ typically lying in the range $\kappa > 2$. Space-craft measurements of electron energy spectra have been successfully modelled with kappa distribution by Vasyliunas [1968] and Christon et al., [1988]. The anisotropic form was first introduced for instability analysis of Jupiter's whistler emission was introduced by Leubner [1982]. Using kappa distribution, Summers and Thorne [1991] have discussed plasma dispersion function theoretically and found general properties of a dielectric tensor for ions and Langmuir waves. Kiran et al., [2005] calculated the power dissipation by obliquely propagating Alfvén wave to heat solar wind protons using non- Maxwellian distribution function and showed the best fit of the results as compared with Maxwellian. Further, the behavior of non- Maxwellian velocity distribution in the field of dusty plasmas has been studied by Rosenberg and Mendis [1992], Lee [2006] as well as Rubab et al. [2006].

Plasma instabilities caused by the field-aligned current have received a lot of attention in the past. The relative streaming of ion/electron component along an ambient magnetic field at a speed greater than the Alfvén speed can excite numerous electromagnetic instabilities, many of which have their largest growth rates with wave vectors aligned along the magnetic field.

The aim of the present paper is to investigate excitation/damping of dust kinetic Alfvén waves by a continuously drifting beam of $\kappa-$ions along the direction of the magnetic field in a magnetized dusty plasma with dust charge fluctuation effects and to highlight the effect of dust parameters on the growth rates of instability.

2 Basic Equations

We consider an electromagnetic dust kinetic Alfvén wave streaming instability in a collisionless electron-ion dusty magnetoplasma. The motion of DKAW is followed by considering thermal and magnetized Lorentzian electrons and ions, cold and magnetized dust ($\omega \ll \omega_{cd}$), and the charge on the dust grain surface is fluctuating with time. We adopt the two potential representations that are commonly used to express elctromagnetic perturbations in the low-beta plasmas to describe the wave electric field. This idea is to decouple the compressional Alfvén mode and consider only the effect of the field line bending. We may neglect the electromagnetic wave compression along the direction of the external magnetic field ($B_{1z} = 0$) and assume two different electrostatic potentials to represent the transverse and parallel components of electric field [Cramer,2001] i.e., $E_\perp = -\nabla_\perp \varphi$ and $E_\parallel = -\partial\psi/\partial z$,where $\varphi \neq \psi$.

The charge quasineutrality condition can be written as $n_{e0} = n_{i0} + Q_{d0}n_{d0}/e$, where $Q_{d0} = \pm Z_{d0}e$

Linearized Poisson's equation is

$$\nabla_\perp^2 \varphi + \frac{\partial^2\psi}{\partial z^2} = 4\pi e\left[n_{e1} - n_{i1} - \frac{Q_{d0}}{e}n_{d1} - \frac{n_{do}}{e}Q_{d1}\right], \tag{1}$$

from Maxwell equation

$$\frac{\partial}{\partial z}\nabla_\perp^2\left(\varphi - \psi\right) = \frac{4\pi}{c^2}\frac{\partial}{\partial t}\left(J_{e1z} + J_{i1z} + J_{d1z}\right), \tag{2}$$

where $J^{'}s$ are the field aligned current densities.

We have assumed electrons ($V_0 = 0$) and ions are non- thermal and strongly magnetized obeying kappa distribution function [Langmayr et al., 2005]

$$f_{j0}^{\kappa} = n_{i0}(\frac{1}{\pi\kappa v_{tj\kappa}^2})^{\frac{3}{2}}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)}\left[1 + \frac{1}{\kappa v_{tj\kappa}^2}\left(v_\perp^2 + (v_\parallel - V_0)^2\right)\right]^{-\kappa-1}; \ \kappa > 3/2, \tag{3}$$

and $v_{tj\kappa}^2 = \left(\frac{2\kappa-3}{\kappa}\right)\left(\frac{T_j}{m_j}\right)$, where κ is the spectral index, Γ is the Gamma function and the thermal speed $v_{tj\kappa}$ is related to the particle temperature T_j.

We have neglected the effect of finite Larmor radius for long perpendicular wavelength as $k_\perp v_{tj} \ll \omega_{cj}$.The perturbed distribution function of $j-$species ($j = e, i$), assuming $k_\perp \rho_j = 0$ is given by the aid of linearized Vlasov equation. The dynamics of cold and magnetized dust is governed by fluid equations.

3 Lorentzian Number Density and Current Density Perturbations

The perturbed number densities of Lorentzian type electrons and ions are found to be

$$n_{e1} = \frac{2e\psi n_{e0}}{m_e v_{te\kappa}}\left[(1 - \frac{1}{2\kappa}) + \zeta_{e0}Z_\kappa(\zeta_{e0})\right], \tag{4}$$

$$n_{i1} = -\frac{2e\psi n_{i0}}{m_i v_{ti\kappa}}\left[(1 - \frac{1}{2\kappa}) + \eta Z_\kappa(\eta)\right], \tag{5}$$

where $Z_\kappa(\eta)$ is the plasma dispersion function for kappa with $\eta = \left(\omega - k_\parallel V_0\right)/(k_\parallel v_{ti\kappa})$ and $\zeta_{e0} = \omega/(k_\parallel v_{te\kappa})$ for non-streaming electrons.

The perturbed number density of cold and magnetized dust grains is given by

$$n_{d1} = \frac{n_{d0}Q_{d0}}{m_d\omega^2}\left[\frac{\omega^2}{\omega^2-\omega_{cd}^2}k_\perp^2\varphi + k_\parallel^2\psi\right]. \tag{6}$$

Now the longitudinal components of current density perturbations are given by

$$J_{e1z} = -\frac{2e^2\psi n_{e0}}{m_e v_{te\kappa}}\zeta_{e0}\left[(1-\frac{1}{2\kappa}) + \zeta_{e0}Z_\kappa(\zeta_{e0})\right], \tag{7}$$

and

$$J_{i1z} = -\frac{2e^2\psi n_{i0}}{m_i v_{ti\kappa}}\left[(1-\frac{1}{2\kappa})\eta + \eta^2 Z_\kappa(\eta)\right]. \tag{8}$$

In the limit $\kappa \to \infty$, our results reduce to their classical Maxwellian counterparts.

The parallel component of perturbed dust current density turns out to be

$$J_{d1z} = \frac{\omega_{pd}^2}{\omega}\varepsilon_0 k_\parallel \psi. \tag{9}$$

3.1 Lorentzian-type Charging Currents

The charging equation containing Lorentzian electrons and ions currents is

$$\frac{\partial Q_{d1}}{\partial t} = \sum I_{e1}^\kappa + I_{i1}^\kappa, \tag{10}$$

where the electron and ion currents which are calculated by using a surface integral through the dust grain surface of radius r_d having potential φ_d are given as

$$I_{e1}^\kappa = -\frac{r_d^2\psi}{4m_e v_{te\kappa}(\lambda_{De\kappa})^2}\frac{\kappa}{(\kappa-1/2)}\left[\zeta_{e0}\left((1-\frac{1}{2\kappa}) + \zeta_{e0}Z_\kappa(\zeta_{e0})\right)m_e v_{te\kappa}^2 + 2e\varphi_d Z_\kappa(\zeta_{e0})\right], \tag{11}$$

and

$$I_{i1}^\kappa = -\frac{r_d^2\psi}{4m_i v_{ti\kappa}(\lambda_{Di\kappa})^2}\frac{\kappa}{(\kappa-1/2)}\left[\eta\left((1-\frac{1}{2\kappa}) + \eta Z_\kappa(\eta)\right)m_i v_{ti\kappa}^2 - 2e\varphi_d Z_\kappa(\eta)\right]. \tag{12}$$

The charge fluctuation on dust grain surface as a function of perturbed potential ψ in the parallel direction is

$$\begin{aligned} Q_{d1} &= \pm\frac{i}{\omega}\Omega\psi \\ \Omega &= \frac{2\kappa}{2\kappa-1}\pi r_d^2\left[\begin{array}{c} \frac{1}{v_{te\kappa}(\lambda_{De\kappa})^2}\left\{\zeta_{e0}\left((1-\frac{1}{2\kappa})+\zeta_{e0}Z_\kappa(\zeta_{e0})\right)m_e v_{te\kappa}^2+2e\varphi_d Z_\kappa(\zeta_{e0})\right\} \\ +\frac{1}{v_{ti\kappa}(\lambda_{Di\kappa})^2}\left\{\eta\left((1-\frac{1}{2\kappa})+\eta Z_\kappa(\eta)\right)m_i v_{ti\kappa}^2-2e\varphi_d Z_\kappa(\eta)\right\} \end{array}\right] \end{aligned} \tag{13}$$

where the symbol ± is used for positively and negatively charged dust grains. Using Eq. (1) and Eq. (2), we obtain the coupled equations as

$$A_1\varphi + B_1\psi = 0, \qquad A_2\varphi + B_2\psi = 0. \tag{14}$$

By applying suitable expansions on plasma dispersion functions, $A_1, A_2, B_{1,}$ and B_2 turns out to be

$$\begin{aligned} A_1 &= k_\perp^2 f_d, \\ A_2 &= k_\parallel k_\perp^2 c^2, \\ B_1 &= \frac{1}{(\lambda_{De}^\kappa)^2} - \frac{\omega_{pd}^2}{\omega^2}k_\parallel^2 - \frac{k_\parallel^2\omega_{pi}^2}{(\omega - k_\parallel V_0)^2}, \\ B_2 &= \frac{\omega^2}{k_\parallel}\frac{1}{(\lambda_{De}^\kappa)^2} - \frac{k_\parallel\omega\omega_{pi}^2}{(\omega - k_\parallel V_0)^2} - k_\parallel k_\perp^2 c^2 - \omega_{pd}^2 k_\parallel, \end{aligned}$$

where $f_d = \omega_{pd}^2/\omega_{cd}^2$.

4 Dispersion Relation

From Eq. (14) we get

$$A_1B_2 - A_2B_1 = 0, \tag{15}$$

which is the dispersion relation of coupled dust kinetic Alfvén and dust acoustic wave in the presence of non-thermal ion beam.

On placing the values of A_1, A_2, B_1,and B_2, we get a six-order equation

$$\omega^6 + P\omega^5 + Q\omega^4 + R\omega^3 + S\omega^2 + T\omega + U = 0, \tag{16}$$

where

$$\begin{aligned}
P &= -2k_\parallel V_0, \\
Q &= k_\parallel^2 \lambda_{De\kappa}^2 {\omega_{pi}}^2 - k_\parallel^2 V_0^2 - k_\parallel^2 \lambda_{De\kappa}^2 (k_\perp^2 c^2 + \omega_{pd}^2) - k_\parallel^2 V_{DA}^2, \\
R &= k_\parallel^3 V_0 \lambda_{De\kappa}^2 {\omega_{pi}}^2 + 2k_\parallel V_0 (k_\perp^2 c^2 + \omega_{pd}^2) + 2k_\parallel^3 V_0 V_{DA}^2, \\
S &= -k_\parallel^2 V_0^2 (k_\perp^2 c^2 + \omega_{pd}^2) - k_\parallel^4 V_0^2 V_{DA}^2 + k_\parallel^4 V_{DA}^2 \lambda_{De\kappa}^2 \left(\omega_{pd}^2 + \omega_{pi}^2\right), \\
T &= -2k_\parallel^5 V_0 V_{DA}^2 \lambda_{De\kappa}^2 \omega_{pd}^2, \\
U &= k_\parallel^6 V_0^2 V_{DA}^2 \lambda_{De\kappa}^2 \omega_{pd}^2,
\end{aligned}$$

where $V_{DA} = c\omega_{cd}/\omega_{pd}$ is the dust Alfvén speed.

By inclusion of dust charge fluctuations effects, the only term B_1 in the dispersion relation will have its effect, i.e.,

$$B_1 = \frac{1}{(\lambda_{De\kappa})^2} - \frac{\omega_{pd}^2}{\omega^2} k_\parallel^2 - \frac{k_\parallel^2 \omega_{pi}^2}{(\omega - k_\parallel V_0)^2} \pm \frac{i r_d^2 n_{d0}}{4} \chi,$$

where

$$\chi = \frac{1}{k_\parallel (\lambda_{Deff\kappa})^2} - \frac{V_0}{\omega (\lambda_{Di\kappa})^2} + 2e\varphi_d k_\parallel \left(\frac{1}{\omega^2 m_i (\lambda_{De\kappa})^2} - \frac{(1 - k_\parallel V_o/\omega)^{-1}}{\omega^2 m_e (\lambda_{Di\kappa})^2} \right),$$

where $\lambda_{Deff\kappa}$ is the effective Debye wavelength for a Lorentzian plasma and is given by [Bryant 1996],

$$\lambda_{Deff\kappa} = \left[\frac{1}{4\pi n e^2} \frac{(\kappa - 3/2)}{(\kappa - 1/2)} \left(\frac{1}{T_e} + \frac{1}{T_i} \right)^{-1} \right]^{1/2}.$$

For $V_0 = 0$, the dispersion relation of Kinetic Alfvén wave in the presence of dust charge fluctuation is given by

$$\omega^2 = k_\parallel^2 V_{DA}^2 \left[1 + \frac{2\kappa - 3}{2\kappa - 1} \left(1 + \frac{\omega_{pd}^2}{k_\perp^2 c^2} \right) k_\perp^2 \rho_d^2 \pm i\pi n_{d0} \frac{r_d^2}{k_\parallel} \right], \tag{17}$$

which is the modified dispersion relation of dust kinetic Alfvén wave with dust charge fluctuation effects. It can be noticed that the first term (MHD Alfvén wave) and the last term (charge fluctuation) remain insensible to the form of distribution function. Dust charge fluctuation is expected to introduce wave damping/growth, which turns out to be very small for the parameters used and thus has not significant effect on the wave properties.

5 Discussions

Low frequency Alfvén waves in dusty plasmas with application to interstellar media [Nakano et al., 2002] have been considered. We discuss the solution of Eq. (16) for several parameters which are close to dusty plasma environments in interstellar clouds.

For computational convenience, we introduce dimensionless parameters as follows

$$\omega = \omega_{cd}\tilde{\omega}, \quad k_{\|} = \frac{\omega_{cd}}{V_{DA}}\tilde{k}_{\|}, \quad V_0 = V_{DA}\tilde{V}_0.$$

In this paper, we have presented the analytical results of dust kinetic Alfvén wave instability with dust charge fluctuation effects based on the Lorentzian distribution function. The presence of a high energy tail (which appears for low values of kappa) reduces the magnitude of the Lorentzian electrons and ions currents. The fact must be due to the variation of charging cross-section at higher energies. Further,the effect of $\kappa-$distributed streaming ions and different dust parameters on the growth rate of instability has been discussed. Using a different form of kappa (anisotropic or loss cone) distribution function may differ the results as presented here.

The dispersion relation for the DKAW instability is found to be dependent on the spectral index κ which means that Lorentzian plasma is able to support a number of unstable branches. Lorentzian index is found to be more effective in the large wavelength limit as compared to the small wavelength where the tail of unstable region remains independent of κ, as shown in Figure 1(a). The effect of angle θ between B_0 and k is shown in Figure 1(b). The growth rates of unstable region decreases as the angle between magnetic field and propagation vector increases and we get maximum growth rates for exact parallel propagation. Figure 1(c) shows that the growth rates change drastically for a very small change in β_d in the region of large wavelength. Ion beam velocity V_0 is observed to have a large impact on the growth rates. The growth rates increase with V_0 as presented in Figure 1d. Charge on the dust grain, Z_d, is found to have a significant effect on the growth rates, i.e., it increases the growth rates, which is depicted in Figure 1(e). When a large number of dust grains are introduced in a plasma, it will enhance the loss rate of electrons by attachment on a dust grain surface which reduces the wave activity. At the same time the electron loss rate increases the drift velocity which in turns helps to excite the DKAWs and a further increase will help to stabilize the system. Effect of number density of dust grains n_d has been shown in Figure 1(f). We have six roots of our dispersion Eq. (16). We get only one root unstable and all other roots remains stable which can easily be seen from Figure 2, where all real and imaginary roots are plotted. It shows that the coupling of the kinetic Alfvén wave and Doppler shifted $\kappa-$ions gives rise to DKAW instability.

The interactions of streaming instabilities with the background turbulence of interstellar medium may suppress the instabilities and cause damping of wave in a nonlinear regime. A penetration of dust from interstellar origin to the vicinity of heliopause could have a significant effect on the wave properties. In particular, field-aligned currents are the major means of ionosphere-magnetosphere coupling and the formation of auroras. Since the change in the auroras is associated with the change in the auroral currents, the

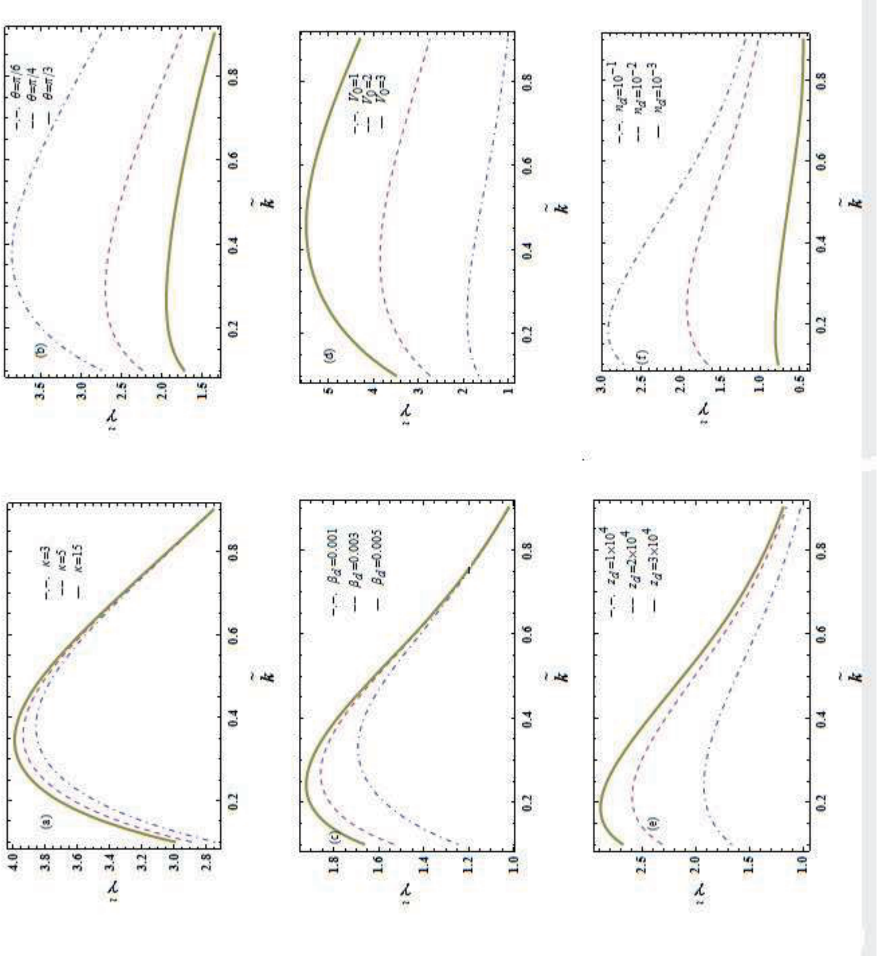

Figure 1: Effect of different parameters on the growth rates of instability

dynamics of Lorentzian-type currents may be an important feature. Measurements from the polar spacecraft show the existence of small scale Alfvén waves that carry a large net poynting flux along magnetic field lines towards the earth. The charged dust may interact with background ionosphere in such a way to produce particle acceleration and auroral like atmospheric emission. Observational data from the Freja wave experiment showed that the transverse scale of the spikes is of the order of inertial/gyroradii scales which is comparable to discrete auroral arc and showed that the Earth auroral plasma energization observed nonlinear soliton-type structures, which can be interpreted as solitary kinetic Alfvén waves. Since the particle distribution function in the auroral source regions with

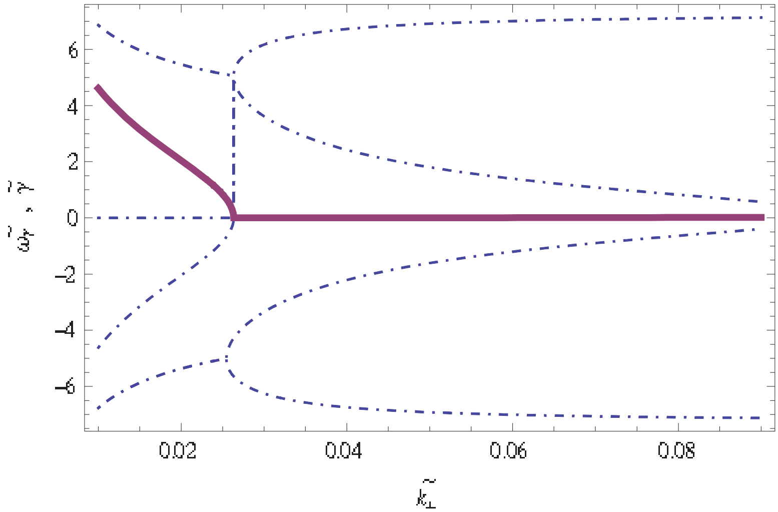

Figure 2: Real(Dashed lines) and Imaginary (solid lines) parts of dispersion equation

high energy tails ($\kappa < 7$)is observed [Wahlund et al., 1994] [Louarn et al., 1994], our results may be useful to study the Lorentzian dust kinetic Alfvén waves in this region. Our results might be important for the particle acceleration by Alfvén waves propagating along the Io flux tube and radio emission in Jupiter.

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