The goal of the DIRAC experiment at CERN (PS212) is to measure the \( \pi^+\pi^- \) atom lifetime with 10% precision. Such a measurement would yield a precision of 5% on the value of the S-wave \( \pi\pi \) scattering lengths combination \(|a_0 - a_2|\). Based on part of the collected data we present a first result on the lifetime, \( \tau = [2.91^{+0.49}_{-0.62}] \times 10^{-15} \) s, and discuss the major systematic errors. This lifetime corresponds to \(|a_0 - a_2| = 0.264^{+0.033}_{-0.020} \) m\(^{-1}\). This article is a short version of the work [2].

1 Introduction

The aim of the DIRAC experiment at CERN [3] is to measure the lifetime of pionium, an atom consisting of a \( \pi^+ \) and a \( \pi^- \) meson (\( A_{2\pi} \)). The lifetime is dominated by the charge-exchange scattering process (\( \pi^+\pi^- \rightarrow \pi^0\pi^0 \)) \(^1\) and is thus related to the relevant scattering lengths [5]. The partial decay width of the atomic ground state (principal quantum number \( n = 1 \), orbital quantum number \( l = 0 \)) is [4, 6, 7]

\[
\Gamma_{1S} = \frac{1}{\tau_{1S}} = \frac{2}{9} \alpha^3 p |a_0 - a_2|^2 (1 + \delta)
\]

with \( \tau_{1S} \) the lifetime of the atomic ground state, \( \alpha \) the fine-structure constant, \( p \) the \( \pi^0 \) momentum in the atomic rest frame, and \( a_0 \) and \( a_2 \) the S-wave \( \pi\pi \) scattering lengths for isospin 0 and 2, respectively. The term \( \delta \) accounts for QED and QCD corrections [7]. It is a known quantity \( (\delta = (5.8\pm1.2) \times 10^{-2}) \) ensuring a 1% accuracy for Eq. (1). A measurement of the lifetime therefore allows to obtain in a model-independent way a value of \(|a_0 - a_2|\).

The \( \pi\pi \) scattering lengths \( a_0, a_2 \) have been calculated within the framework of Standard Chiral Perturbation Theory [8] with a precision better than 2.5% [9] (\( a_0 = 0.220 \pm 0.005, a_2 = -0.0444 \pm 0.0010, a_0 - a_2 = 0.265 \pm 0.004 \) in units of inverse pion mass) and lead to the prediction \( \tau_{1S} = (2.9 \pm 0.1) \times 10^{-15} \) s. Generalized Chiral Perturbation Theory though allows for larger \( a \)-values [10]. Model independent measurements of \( a_0 \) have been done using \( K_{e4} \) decays [11, 12].

\(^1\)Annihilation into two photons amounts to \( \approx 0.3\% \) [4] and is neglected here.
Oppositely charged pions emerging from a high energy proton-nucleus collision may be either produced directly or stem from strong decays ("short-lived" sources) and electromagnetic or weak decays ("long-lived" sources) of intermediate hadrons. Pion pairs from "short-lived" sources undergo Coulomb final state interaction and may form atoms. The region of production being small as compared to the Bohr radius of the atom and neglecting strong final state interaction, the cross section $\sigma^s_A$ for production of atoms with principal quantum number $n$ is related to the inclusive production cross section for pion pairs from "short lived" sources without Coulomb correlation ($\sigma^0_s$) [13]:

$$\frac{d\sigma^s_A}{d\vec{p}_A} = (2\pi)^3 \frac{E_A}{M_A} |\Psi^C_n(r^* = 0)|^2 \frac{d^2\sigma^0_s}{d\vec{p}_+ d\vec{p}_-} |_{\vec{p}_+ = \vec{p}_-}$$

with $\vec{p}_A$, $E_A$ and $M_A$ the momentum, energy and mass of the atom in the lab frame, respectively, and $\vec{p}_+$, $\vec{p}_-$ the momenta of the charged pions. The square of the Coulomb atomic wave function for zero distance $r^*$ between them in the c.m. system is $|\Psi^C_n(0)|^2 = \frac{p_B}{\pi n^3}$, where $p_B = m_\pi \alpha/2$ is the Bohr momentum of the pions and $m_\pi$ the pion mass. The production of atoms occurs only in $S$-states [13].

Final state interaction also transforms the “unphysical” cross section $\sigma^0_s$ into a real one for Coulomb correlated pairs, $\sigma_C$ [14, 15]:

$$\frac{d^2\sigma_C}{d\vec{p}_+ d\vec{p}_-} = |\Psi^C_{-k^*}(r^*)|^2 \frac{d^2\sigma^0_s}{d\vec{p}_+ d\vec{p}_-},$$

where $\Psi^C_{-k^*}(r^*)$ is the continuum wave function and $2k^* \equiv \vec{q}$ with $\vec{q}$ being the relative momentum of the $\pi^+$ and $\pi^-$ in the c.m. system. $|\Psi^C_{-k^*}(r^*)|^2$ describes Coulomb correlation and at $r^* = 0$ coincides with the Gamov-Sommerfeld factor $A_C(q)$ with $q = |\vec{q}|$ [15]:

$$A_C(q) = \frac{2\pi m_\pi \alpha/q}{1 - \exp(-2\pi m_\pi \alpha/q)}.$$  

For low $q$, $0 \leq q \leq q_0$, Eqs. (2, 3, 4) relate the number of produced $A_{2\pi}$ atoms, $N_A$, to the number of Coulomb correlated pion pairs, $N_{CC}$ [16]:

$$\frac{N_A}{N_{CC}} = \frac{\sigma^\text{tot}_A}{\sigma^\text{tot}_C \bigg|_{q \leq q_0}} = \frac{(2\pi \alpha m_\pi)^3}{\pi} \sum_{n=1}^{\infty} \frac{1}{n^3} \frac{1}{q_0} \int_0^{q_0} A_C(q) d^3q = k_{th}(q_0).$$

In order to account for the finite size of the pion production region and of the two-pion final state strong interaction, the squares of the Coulomb wave functions in Eqs. (2) and (3) must be substituted by the square of the complete wave functions, averaged over the distance $r^*$ and the additional contributions from $\pi^0 \pi^0 \rightarrow A_{2\pi}$ as well as $\pi^0 \pi^0 \rightarrow \pi^+ \pi^-$ [15]. It should be noticed that these corrections essentially cancel in the $k$-factor (Eq. 4). For the sake of clarity we use the symbol $Q$ for the reconstructed and $q$ for the physical relative momentum.
and lead to a correction of only a fraction of a percent. Thus finite size corrections can safely be neglected for $k_{th}$.

Once produced, the $A_{2\pi}$ atoms propagate with relativistic velocity (average Lorentz factor $\bar{\gamma} \approx 17$ in our case) and, before they decay, interact with target atoms, whereby they become excited/de-excited or break up. The $\pi^+\pi^-$ pairs from break-up (atomic pairs) exhibit specific kinematical features which allow to identify them experimentally [13], namely very low relative momentum $q$ and $q_L$ (the component of $\vec{q}$ parallel to the total momentum $\vec{p}_+ + \vec{p}_-$). After break-up, the atomic pair traverses the target and to some extent loses these features by multiple scattering, essentially in the transverse direction, while $q_L$ is almost not affected. This is one reason for analyzing the data in $Q_L$ as well as in $Q$.

Excitation/de-excitation and break-up of the atom are competing with its decay. Solving the transport equations with the cross sections for excitation and break-up, [18] leads to a target-specific relation between break-up probability and lifetime which is estimated to be accurate at the 1% level [19]. Measuring the break-up probability thus allows to determine the lifetime of pionium [13].

The first observation of the $A_{2\pi}$ atom [20] has allowed to set a lower limit on its lifetime [16, 17] of $\tau > 1.8 \times 10^{-15}$ s (90% CL). In this paper we present a determination of the lifetime of the $A_{2\pi}$ atom, based on a large sample of data taken in 2001 with Ni targets.

2 The DIRAC experiment

The DIRAC experiment uses a magnetic double-arm spectrometer at the CERN 24 GeV/c extracted proton beam T8. Details on the set-up may be found in [21]. Since its start-up, DIRAC has accumulated about 15’000 atomic pairs. The data used for this work were taken with two Ni foils, one of 94$\mu$m thickness (76% of the $\pi^+\pi^-$ data), and one of 98$\mu$m thickness (24% of the data). An extensive description of the DIRAC set-up, data selection, tracking, Monte Carlo procedures, signal extraction and a first high statistics demonstration of the feasibility of the lifetime measurement, based on the Ni data of 2001, have been published in [22]. The set-up and the definitions of detector acronyms are shown in Fig. 1.

Pairs of oppositely charged pions are selected by means of Cherenkov, preshower and muon counters. Through the measurement of the time difference between the vertical hodoscope signals of the two arms, time correlated (prompt) events ($\sigma_{\Delta t} = 185$ ps) can be distinguished from accidental events (see [22]). The resolution of the three components of the relative momentum $Q$ of two tracks, transverse and parallel to the c.m. flight direction, $Q_x$, $Q_y$ and $Q_L$, is about 0.5 MeV/c for $Q \leq 4$MeV/c. Due to charge combinatorials and inefficiencies of the SFD, the distributions for the transverse components have substantial tails, which the longitudinal component does not exhibit [23]. This is yet another reason for analyzing the data both in $Q$ and $Q_L$. Data were analyzed with the help of the DIRAC analysis software package ARIANE [25].
3 Analysis

The spectrometer including target is fully simulated by GEANT-DIRAC [24], a GEANT3-based simulation code. The detectors, including read-out, inefficiency, noise and digitalization are simulated and implemented into the DIRAC analysis code ARIANE [25]. The triggers are fully simulated as well. The Monte Carlo simulation of different event types and their reconstruction produce distributions with exactly the same procedures and cuts as used for experimental data. The different event types are generated according to the underlying physics.

Atomic pairs: The atomic $\pi^+\pi^-$ pairs are generated according to the probabilities and kinematics described by the evolution of the atom while propagating through the target and by the break-up process (see [26]). Pairs total momenta are generated according to the measured total momentum distributions for short-lived pairs. These $\pi^+\pi^-$ pairs, starting from their spatial production point, are then propagated through the remaining part of the target and the full spectrometer using GEANT-DIRAC. Reconstruction of the track pairs using the fully simulated detectors and triggers leads to the atomic pair distribution $dn_{MC}^{A}/dQ$.

Coulomb correlated $\pi^+\pi^-$ pairs (CC-background): The events are generated according to Eqs. (3,4) using measured total momentum distributions for short-lived pairs. The generated $q$-distributions are assumed to follow phase space modified by the Coulomb correlation function (Eq. (4)), $dN_{CC}^{gen}/dq \propto q^2 \times A_C(q)$. Processing them with GEANT-DIRAC and then analyzing them using full detector and trigger simulation leads to the Coulomb correlated distribution $dN_{MC}^{CC}/dQ$. 

Figure 1: Schematic top view of the DIRAC spectrometer. Upstream of the magnet: target, microstrip gas chambers (MSGC), scintillating fiber detectors (SFD), ionization hodoscopes (IH) and iron shielding. Downstream of the magnet: drift chambers (DC), vertical and horizontal scintillation hodoscopes (VH, HH), gas Cherenkov counters (Ch), preshower detectors (PSh) and, behind the iron absorber, muon detectors (Mu).
Non-correlated $\pi^+\pi^-$ pairs (NC-background): $\pi^+\pi^-$ pairs, where at least one pion originates from the decay of a "long-lived" source (e.g. electromagnetically or weakly decaying mesons or baryons) do not undergo any final state interaction. Thus they are generated according to $dN_{\text{NC}}^{\text{gen}}/dq \propto q^2$, using slightly softer momentum distributions than for short-lived sources (difference obtained from FRITIOF-6). The Monte Carlo distribution $dN_{\text{NC}}^{\text{MC}}/dq$ is obtained as above.

Accidental $\pi^+\pi^-$ pairs (acc-background): $\pi^+\pi^-$ pairs, where the two pions originate from two different proton-nucleus interactions, are generated according to $dN_{\text{acc}}^{\text{gen}}/dq \propto q^2$, using measured momentum distributions. The Monte Carlo distribution $dN_{\text{acc}}^{\text{MC}}/dq$ is obtained as above.

All the Monte Carlo distributions are normalized, $\int_0^{Q_{\text{max}}}(dN_{i}^{\text{MC}}/dq)dQ = N_i^{\text{MC}}$, $i = A, CC, NC, acc$. The measured prompt distributions are approximated by appropriate functions. The functions for atomic pairs, $F_A(Q)$, and for the backgrounds, $F_B(Q)$, (analogously for $Q_L$) are defined as:

$$
F_A(Q) = \frac{n_A^{\text{rec}} dN_A^{\text{MC}}}{n_A^{\text{MC}} dQ} + \frac{N_{CC}^{\text{MC}}}{N_{CC}^{\text{MC}} dQ} + \frac{N_{NC}^{\text{MC}}}{N_{NC}^{\text{MC}} dQ} + \frac{\omega_{\text{acc}} N_{\text{pr}}}{N_{\text{acc}}^{\text{MC}} dQ}
$$

with $n_A^{\text{rec}}$, $N_{CC}^{\text{MC}}$, $N_{NC}^{\text{MC}}$ the reconstructed number of atomic pairs, Coulomb- and non-correlated background, respectively, and $\omega_{\text{acc}}$ the fraction of accidental background out of all prompt events $N_{\text{pr}}$. Analyzing the time distribution measured with the vertical hodoscopes (see [22]) we find $\omega_{\text{acc}} = 7.1\%$ ($7.7\%$) for the 94 $\mu$m (98 $\mu$m) data sets [22, 23] and keep it fixed when fitting. The measured distributions as well as the background are shown in Fig. 2 (top).

First, we determine the background composition by minimizing $\chi^2$ outside of the atomic pair signal region, i.e. for $Q > 4\text{MeV}/c$ and $Q_L > 2\text{MeV}/c$. For this purpose we require $n_A^{\text{rec}} = 0$. As a constraint, the background parameters $N_{CC}^{\text{MC}}$ and $N_{NC}^{\text{MC}}$ representing the total number of CC- and NC-events, have to be the same for $Q$ and $Q_L$. Then, with the parameters found, the background is subtracted from the measured prompt distribution, resulting in the residual spectra. For the signal region, defined by the cuts $Q = 4\text{MeV}/c$ and $Q_L = 2\text{MeV}/c$, we obtain the total number of atomic pairs, $n_A^{\text{residual}}$ and of Coulomb correlated background events, $N_{CC}^{\text{sig}}$. Results of fits for $Q$ and $Q_L$ together are shown in Table 1. CC-background and NC- or acc-backgrounds are distinguishable due to their different shapes, most pronounced in the $Q_L$ distributions (see Fig. 2). Accidental and NC-background shapes are almost identical for $Q$ and fully identical in $Q_L$.

Second, the atomic pair signal may be directly obtained by fitting distributions over the full range and including the Monte Carlo shape distribution $F_A$ ("shape fit"). The signal strength has to be the same in $Q$ and $Q_L$. The result for the signal strength $n_A^{\text{rec}}$ as well as the CC-background below the cuts, $N_{CC}^{\text{sig}}$, are shown in Table 1.

In order to deduce the break-up probability, $P_{\text{br}} = n_A/N_A$, the total number of atomic pairs $n_A$ and the total number of produced $A_{2\pi}$ atoms, $N_A$, have to be known. The procedure of obtaining the two quantities requires reconstruction efficiencies and relies on the known relation between produced atoms and Coulomb correlated $\pi^+\pi^-$ pairs of.
Figure 2: Top: Experimental $Q$ and $Q_L$ distributions after subtraction of the prompt accidental background, and fitted Monte Carlo backgrounds (dashed lines). The peak at $Q = 4 \text{ MeV}/c$ is due to the cut $Q_T \leq 4 \text{ MeV}/c$. Bottom: Residuals after background subtraction. The dashed lines represent the expected atomic signal shape. The bin-width is 0.25 MeV/c.

Table 1: Break-up probabilities for the combined Ni2001 data, based on the fit results and the $k$-factors for the cuts $Q_{\text{cut}} = 4 \text{MeV}/c$ and $Q_{L,\text{cut}} = 2 \text{MeV}/c$.

<table>
<thead>
<tr>
<th>$Q$</th>
<th>$n_A^{\text{residual}}$</th>
<th>$n_A^{\text{rec}}$</th>
<th>$N_{CC}^{\text{rec}}$</th>
<th>$k$</th>
<th>$P_{br}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$</td>
<td>6518 ± 373</td>
<td>106500 ± 1130</td>
<td>0.1384</td>
<td>0.442 ± 0.026</td>
<td></td>
</tr>
<tr>
<td>$Q_L$</td>
<td>6509 ± 330</td>
<td>82289 ± 873</td>
<td>0.1774</td>
<td>0.445 ± 0.023</td>
<td></td>
</tr>
<tr>
<td>$Q &amp; Q_L$</td>
<td>6530 ± 294</td>
<td>106549 ± 1004</td>
<td>0.447 ± 0.023</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Eq. (5). The break-up probability $P_{br}$ thus becomes:

$$P_{br} = \frac{n_A}{N_A} = \frac{n_A^{\text{rec}}(Q \leq Q_{\text{cut}})}{k(Q_{\text{cut}})N_{CC}^{\text{rec}}(Q \leq Q_{\text{cut}})} \text{ with } k(Q_{\text{cut}}) = \frac{k_{th}(q_0)}{\epsilon_{CC}/\epsilon_A}.$$  

where $\epsilon_A$ and $\epsilon_{CC}$ are reconstruction efficiencies for atomic pairs and CC-pairs. The break-up probabilities from $Q$ and $Q_L$ agree within a fraction of a percent (see Table 1). The values from shape fit and from background fit are in perfect agreement as compared to the statistical error. As a result we adopt $P_{br} = 0.447 \pm 0.023_{\text{stat}}$.

The break-up probability has to be corrected for the impurities of the targets. The 94 $\mu$m thick target has a purity of only 98.4%, while the 98 $\mu$m thick target is 99.98% pure. The impurities being mostly of smaller atomic number than Ni lead for both targets together to a reduction of break-up probability of 1.1% as compared to pure Ni. Thus the measured break-up probability has to be increased by 0.005 in order to correspond to
pure Ni, with the final result of:

\[ P_{br} = 0.452 \pm 0.023_{\text{stat}} \]  

(8)

4 Systematic errors

The difference in break-up probability while varying fit range leads to \( \Delta P_{br}^{\text{range}} = 0.023 \). Consistency of the procedure requires that the break-up probability does not depend on \( Q_{\text{cut}} \). There is a systematic effect which, however, levels off for large cut momenta: the more the signal is contained in the cut, the more the \( P_{br} \) values stabilize. This may be due to systematics in the atomic pair shape directly and/or in reconstructed CC-background for small relative momenta. This was investigated by assuming two extreme models for atom break-up: the break-up occurs either only from the 1S-state or from highly excited states. The two extremes result in a difference in break-up probability of \( \Delta P_{br}^{\text{shape}} = 0.008 \).

We have investigated possible uncertainties in multiple scattering as simulated by GEANT by changing the scattering angle in the GEANT simulation by \( \pm 5\% \). As a result, the break-up probability changes by 0.002 per one percent change of multiple scattering angle. In fact we have measured the multiple scattering for all scatterers (upstream detectors, vacuum windows, target) and found narrower angular distributions than expected from the standard GEANT model [27]. This, however, may be due also to errors in determining the thickness and material composition of the upstream detectors. Based on these studies we conservatively attribute a maximum error of \( +5\% \) and \( -10\% \) to multiple scattering.

Another source of uncertainty may be due to the presence of unrecognized \( K^+K^- \) and \( \bar{p}p \) pairs that would fulfill all selection criteria [28]. Such pairs may be as abundant as 0.5\% and 0.15\%, respectively, of \( \pi^+\pi^- \) pairs as estimated for \( K^+K^- \) with FRITIOF-6 and for \( \bar{p}p \) from time-of-flight measurements in a narrow momentum interval with DIRAC data. Their mass renders the Coulomb correlation much more peaked at low \( Q \) than for pions, which leads to a change in effective \( \pi^+\pi^- \) Coulomb background at small \( Q \), thus to a smaller atomic pair signal and therefore to a decrease of break-up probability. The effect leads to a change of \( \Delta P_{br}^{K\bar{K}, \bar{p}p} = -0.04 \). We do not apply this shift but consider it as a maximum systematic error of \( P_{br} \). Admixtures from unrecognized \( e^+e^- \) pairs from photon conversion do not contribute because of their different shapes.

Finally, the correlation function Eq. (3) used in the analysis is valid for pointlike production of pions, correlated only by Coulomb final state interaction (Eq. (4)). However, there are corrections due to finite size and strong interaction [15]. These have been studied based on the UrQMD transport code simulations [29] and DIRAC data on \( \pi^-\pi^- \) correlations. The corrections lead to a change of \( \Delta P_{br}^{\text{finite-size}} = -0.02 \). Due to the uncertainties we conservatively consider 1.5 times this change as a maximum error, but do not modify \( P_{br} \).

The systematics are summarized in Table 2. The extreme values represent the ranges of the assumed uniform probability density function (u.p.d.f.), which, in case of asymmetric errors, were complemented symmetrically for deducing the corresponding standard deviations \( \sigma \). Convoluting the five u.p.d.f. results in bell-shaped curves very close to a
Table 2: Summary of systematic effects on the measured break-up probability $P_{br}$.

<table>
<thead>
<tr>
<th>source</th>
<th>extreme values</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC-background</td>
<td>$+0.012/ -0.012$</td>
<td>$\pm0.007$</td>
</tr>
<tr>
<td>signal shape</td>
<td>$+0.004/ -0.004$</td>
<td>$\pm0.002$</td>
</tr>
<tr>
<td>multiple scattering</td>
<td>$+0.01/ -0.02$</td>
<td>$+0.006$</td>
</tr>
<tr>
<td>$K^+K^-$ and $\bar{p}p$</td>
<td>$+0/ -0.04$</td>
<td>$+0$</td>
</tr>
<tr>
<td>finite size</td>
<td>$+0/ -0.03$</td>
<td>$-0.017$</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>$+0.009/ -0.032$</strong></td>
<td></td>
</tr>
</tbody>
</table>

Gaussian, and the $\pm\sigma$ (Table 2, total error) correspond roughly to a 68.5% confidence level and can be added in quadrature to the statistical error.

The final value of the break-up probability is

$$P_{br} = 0.452 \pm 0.023^{\text{stat}} +0.009_{-0.032}^{\text{syst}} = 0.452 \pm 0.025_{-0.039}.$$  \hfill (9)

5 Lifetime of Pionium

The lifetime may be deduced on the basis of the relation between break-up probability and lifetime for a pure Ni target (Fig. 3). This relation, estimated to be accurate at the 1% level, may itself have uncertainties due to the experimental conditions. Thus the target thickness is estimated to be correct to better than $\pm 1 \mu m$, which leads to an error in the lifetime less than 1% of the expected lifetime and negligible. The result for the lifetime is

$$\tau_{1S} = \left[2.91^{+0.45}_{-0.38} \text{stat} +0.19_{-0.49} \text{syst}\right] \times 10^{-15} \text{s} = \left[2.91^{+0.49}_{-0.62}\right] \times 10^{-15} \text{s}. \hfill (10)$$

Figure 3: Break-up probability $P_{br}$ as a function of the lifetime of the atomic ground state $\tau_{1S}$ for the Ni targets. The experimentally determined $P_{br}$ with statistical and total errors translates into a value of the lifetime with corresponding errors.
The errors are not symmetric because the $P_{br} - \tau$ relation is not linear, and because finite size corrections and heavy particle admixtures lead to possible smaller values of $P_{br}$. With full statistics (2.3 times more than analysed here) the statistical errors may be reduced accordingly. The two main systematic errors (particle admixtures and finite size correction) will be studied in more detail in the future program of DIRAC.

Using Eq. (1), the above lifetime corresponds to $|a_0 - a_2| = 0.264^{+0.033}_{-0.020}$ m$^{-1}$.

References


[2] B. Adeva et al., “First lifetime measurement of the $\pi^+\pi^-$ atom”, hep-ex/0504044; to be published.


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