# A FLY OFF OF THE FAST ELECTRON FLOWS GENERATING TYPE III BURSTS

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**Abstract**

Recent observation data show the existence of a fine structure in solar type III bursts. A solar burst consists of several impulses of radioemission with a short duration - much shorter than burst duration. These impulses are thought to be generated by the different electron beams propagating in the solar corona. It is important to find out the variation electron distribution function taking into account that electrons emit and absorb plasma waves. An initial electron distribution function is taken as a set of mono-energetic beams $f_o(v) = \sum_{i=1}^{n} n_i \delta(v - u_i)$. Then the spread of electrons is described by the propogation of mono-energetic beams. Recently it has been shown that a mono-energetic beam propagates with constant velocity in the form of a beam-plasma structure that consists of electrons and plasmons. The resultant electron distribution function in every point is determined as a result of the beam-plasma structure interaction. Such an approach allows us to obtain the electron distribution function and the spectral energy density of Langmuir waves. The electron distribution function looks like a common plateau and a staircase. The spectral energy density of plasma waves turns to be equal to zero at $v = u_i$ and the respective stair $(u_i < v < u_{i+1})$ is separated from the common plateau. Thus the maximum velocity of the common plateau is decreased jumping down from $v = u_{i+1}$ to $v = u_i$ but its height is increased. For velocities $v$, which are greater than the maximum velocity $u_i$, the electron distribution function has a staircase-form with stair heights decreasing with velocity. The solutions of the problem obtained in the paper are compared with numerical simulations of spreading of electron flows which generate type III bursts.

## 1 Introduction

The observations and theoretical reasons force to search for a satisfactory model of the type III solar bursts [Slottje, 1980; Raoult, 1980; Dulk et al., 1985]. To achieve this goal we have to describe the emission observed as well as propagation of electron beams

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which are supposed to generate this radiation. These electron beams are accelerated in the low atmosphere and propagate along opened magnetic field lines through corona up to the Earth's orbit. The electrons propagating through the coronal plasma cause the generation of Langmuir waves which, in turn, influence the electron spread. It is necessary to point out that the conditions in the corona are such that the quasi-linear time is much smaller then the time of flying-off ($\tau_{qv} \approx (\omega_p n_b/n)^{-1} \ll t$). As it was shown in numerical simulations [Takakura, 1982] the electron beam propagation is nearly one dimensional. Earlier this problem was considered in the paper by Rutov and Sagdeev [1970], where quasi-gasdynamic equations were obtained. Later the full system of gas-dynamic equations for one mono-energetic beam was found [Mel'nik, 1990; Mel'nik, 1995]. Observations carried out at the starting frequency of solar type III bursts [Benz et al., 1982] and well correlated in time solar bursts at the higher frequencies up to the hard X-ray emission with standard type III bursts [Carreia and Kaufmann, 1987] show the fine time structure of solar radiation. This fine structure of the radiation (spikes) could be generated by the set of electron beams.

The main aim of presented paper is to find a proper one dimensional description of electron beam propagation. Due to the smallness of the quasi-linear time we use a gas-dynamic description. Integrating the kinetic quasi-linear equations the system of gas-dynamic equations is obtained. Using these equations the solution for a mono-energetic beam is found when the initial spatial distribution of electrons is given. The plasma waves generated by the propagating electrons force electrons together with plasmons to be concentrated near the electron mean velocity and to move as a beam-plasma structure with constant speed. When the electron distribution function of the source consists of two beams, also the solution commonly consits of two "interacting" beam-plasma structures. However, for small densities of the slower beam only one beam-plasma structure is formed. Also the electron distribution function initially consisting of two plateaus (two steps on the electron distribution function) is converted into only one plateau at some areas of $(x, t)$-plane. $N$ mono-energetic beams propagate as a row of "interacting" beam-plasma structures. Again, in some areas of $(x, t)$-plane the plateaus unite into the common plateau starting from the minimum velocity that leads total number of beam-plasma structures to be decreased.

The one beam propagation is considered in parts 2 and 3; part 2 deals with the main gas-dynamic equations and part 3 with the respective solutions for the time-independent particle source. The two beam dynamic is discussed in sections 4 and 5. In section 6 we consider the dynamics of many electron beams. The main results and the beam flying-off properties are presented in conclusion.

## 2 Gas-dynamic equations for one mono-energetic beam

Let the mono-energetic electron source be placed near $x = 0$ and let it be switched on at $t = 0$. Then we have

$$f(v, x, t = 0) = n\delta(v - u_0)exp(-|x|/d) \ . \quad (1)$$

We also assume that number density of the electron beam is much smaller than the number plasma density ($n \ll n_p$-number plasma density). So in the theory of weak turbulence the electron beam propagation is described by the quasi-linear equations. For more interesting cases the spontaneous terms on the right side of the equations are omitted because they are much smaller than the induced terms. Also the group velocity of Langmuir waves is small, as $v_{gr} \approx v_{Te}^2/v \ll v$. Thus the spatial transfer of energy by plasmons vanishes and this term can be omitted. In one dimensional case the quasi-linear equations are

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} = \frac{4\pi^2 e^2}{m^2}\frac{\partial}{\partial v}\frac{W}{v}\frac{\partial f}{\partial v} \tag{2}$$

$$\frac{\partial W}{\partial t} = \frac{\pi\omega_p}{n}v^2 W\frac{\partial f}{\partial v}\ , \tag{3}$$

where $W(v,x,t)$ is the spectral energy density of Langmuir waves, $f(v,x,t)$ the electron distribution function and $\omega_p = kv$. It is well known that in uniform problem the electron beam relaxes to the plateau steady state for the quasi-linear time. In the zero-order approximation of the gas-dynamic consideration [Rutov and Sagdeev, 1970] the steady state is formed in every spatial point. The electron distribution function $f_s(v,x,t)$ and the spectral energy density $W_s(v,x,t)$, which make the right hand side of the equations vanish, look like

$$f_s(v,x,t) = \begin{cases} p(x,t), & v < u(x,t) \\ 0, & v > u(x,t) \end{cases} \tag{4}$$

$$W_s(v,x,t) = \begin{cases} W(v,x,t), & v < u(x,t) \\ 0, & v > u(x,t) \end{cases}\ , \tag{5}$$

where $p(x,t)$ is the plateau height and $u(x,t)$ the maximum velocity. $p(x,t)$ and $u(x,t)$ are parameters which play the same role as velocity, density and temperature do in the ordinary gasdynamic theory.

Substituting equations (4) and (5) into (2) and integrating over the vicinity of $v = u(x,t)$, we obtain the equation for maximum velocity

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0\ , \tag{6}$$

Whereby $p(x,t)$ satisfies the continuity equation, i.e.,

$$\frac{\partial p}{\partial t} + \frac{u}{2}\frac{\partial p}{\partial x} = 0\ . \tag{7}$$

Using equations (2), (3) we have

$$\frac{\partial f_s}{\partial t} + v\frac{\partial f_s}{\partial x} = \frac{\omega_p}{m}\frac{\partial}{\partial v}\frac{1}{v^3}\frac{\partial W_s}{\partial t}\ . \tag{8}$$

Substituting equation (4) into (8) we obtain

$$\frac{\partial p}{\partial t} + v\frac{\partial p}{\partial x} = \frac{\omega_p}{m}\frac{\partial}{\partial v}\frac{1}{v^3}\frac{\partial W}{\partial t},\ 0 < v < u(x,t)\ . \tag{9}$$

The conditions for $W(v,x,t)$ at $v = u(x,t)$ can be obtained by integrating in the vicinity of $v = u(x,t)$ both equation (8) and equation (8) multiplied by $v$. As a result the two conditions are

$$\frac{\partial W}{\partial t} = 0, \;\; v = u(x,t) \tag{10}$$

$$\frac{\partial u}{\partial t} W = 0, \;\; v = u(x,t) \; . \tag{11}$$

The equations (6), (7), (9), (10), (11) thus obtained are the system of gas-dynamic equations describing electron beam flying-off.

## 3 Solution of gas-dynamic equations for a mono-energetic beam

As is well known [Vedenov and Rutov, 1972], at every instance of time of quasi-linear relaxtion, steady state is characterised by $W|_{v=u} \neq 0$, so that we obtain from condition (11)

$$\frac{\partial u}{\partial t} = 0 \; . \tag{12}$$

Consequently both equations (12) and (6) give that

$$u(x,t) = const = u_0 \; . \tag{13}$$

Then it is easy to obtain from equations (6), (7) and (9) to (11) involving initial condition (1)

$$p(x,t) = n \cdot exp(-|x - ut/2|/d) \tag{14}$$

$$W(v,x,t) = \frac{m}{\omega_p} v^4 (1 - \frac{v}{u}) p(x,t) + \frac{m}{\omega_p} v^3 \phi(x,v) \; . \tag{15}$$

The surrounding plasma is supposed not to be strongly turbulent ($W_T \ll W$)($W_T$-thermal level of plasma oscillations). Therefore at the time $t = 0$ spectral energy density of plasma waves is determined by the initial conditions and quasi-linear relaxation $W \approx v^4$, that together with equation (15) gives

$$\phi(x,v) = \frac{v^2}{u} p(-|x|/u) \; . \tag{16}$$

The changing plateau height is shown in Figure 1. It can be seen that the maximum number of electrons moves with velocity $v = u/2$. In accordance with equation (15) the maximum of the spectral energy density of Langmuir waves is situated at the straight line $x = ut/2$ and at $x = 0$. The first maximum (at $x = ut/2$) corresponds to the moving beam-plasma structure.

The concentration of fast electrons and plasmons along the line $x = ut/2$ is connected with the fact that at the forward flow front ($x > ut/2$) electrons with relatively large velocities (distribution function $\partial f/\partial v > 0$) chiefly come into the given point. During the fast processes of quasi-linear relaxation part of the energy transits from electrons to plasmons

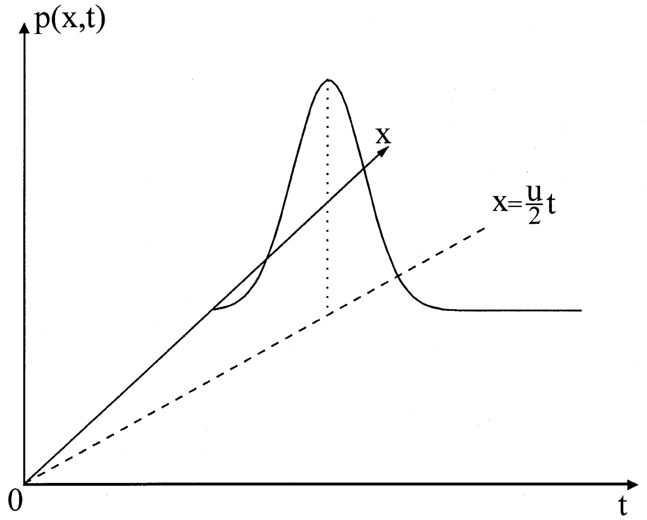



*Figure 1: Plateau height changing in space and time.*

so that electrons are slowed down. At the back front ($x < ut/2$) vice versa, electrons with relatively small velocities come into majority with ($\partial f/\partial v < 0$) that leads plasma oscillations to be absorbed. As a result plasmons and electrons accumulate near line $x = ut/2$. Processes of plasmon emission and absorption force particles to propagate as a beam-plasma structure with constant velocity $u/2$. Moreover the shape of beam-plasma structure does not change with time and space and is determined by the source.

## 4 The main equations for two beam source

In case the initial electron distribution function contains two beams and again the particles are placed near $x = 0$, we have

$$f(v, x, t = 0) = (n_1\delta(v - u_{10}) + n_2\delta(v - u_{20}))exp(-|x|/d) \ . \tag{17}$$

The uniform solution of the two beam relaxation problem (without propagation term in equation (2)) consists of two possible steady states. In particular the state is determined by the initial beam number densities. The electron distribution function consists of two steps and $W_s$ equals to zero at $v = u_{10}$ (Figure 2) when $n_1/u_{10} > n_2/(u_{20} - u_{10})$

$$f_s = \begin{cases} \frac{n_1}{u_{10}}, & v < u_{10} \\ \frac{n_2}{u_{20} - u_{10}}, & u_{10} < v < u_{20} \\ 0, & v > u_{20} \end{cases} \tag{18}$$

$$W_s = \frac{mv^3}{\omega_p} \begin{cases} \frac{n_1}{u_{10}} v, & v < u_{10} \\ \frac{n_2}{u_{20} - u_{10}}(v - u_{10}), & u_{10} < v < u_{20} \\ 0, & v > u_{20} \end{cases} \tag{19}$$

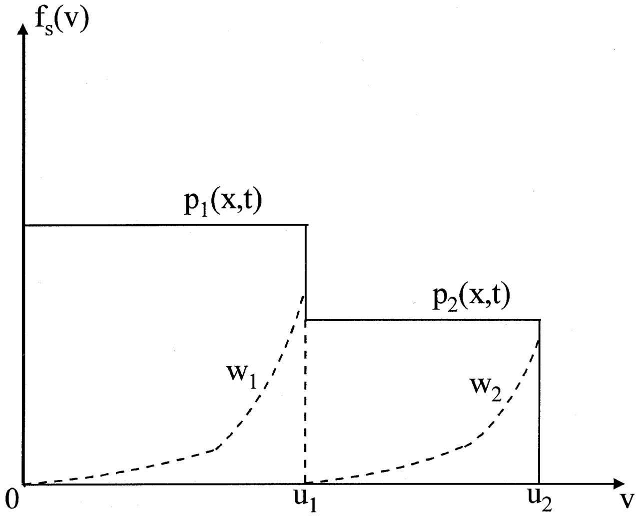



*Figure 2: Steady state with two plateaus in the electron distribution function and corresponding spectral energy density of plasma waves - dashed line.*

In case $n_1/u_{10} < n_2/(u_{20} - u_{10})$ the electron distribution function is just a plateau, but the spectral energy density of Langmuir waves has a drop at $v = u_{10}$ because of slow beam presence

$$f_s = \begin{cases} \dfrac{n_1 + n_2}{u_{20}}, & v < u_{10} \\ 0, & v > u_{20} \end{cases} \tag{20}$$

$$W_s = \frac{mv^3}{\omega_p} \begin{cases} \dfrac{n_1 + n_2}{u_{20}} v, & v < u_{10} \\ \dfrac{n_1 + n_2}{u_{20}} v - n_1, & u_{10} < v < u_{20} \\ 0, & v > u_{20} \end{cases} . \tag{21}$$

The electron source initially consisting of two beams can form the only plateau state in some area of $(x, t)$-plane. This is caused by the difference of electron velocities and number densities of beams. Also electrons of one beam source propagating in plasma can create two steps in the electron distribution function because electrons with near maximum velocity leave source region faster than slow particles. Thus the electron distribution function in the areas of $(x, t)$-plane where it has two steps is considered to be

$$f_s(v, x, t) = \begin{cases} p_1(x, t), & v < u_1(x, t) \\ p_2(x, t), & u_1(x, t) < v < u_2(x, t) \\ 0, & v > u_2(x, t) \end{cases} \tag{22}$$

and spectral energy density

$$W_s(v, x, t) = \begin{cases} W_1(x, t), & v < u_1(x, t) \\ W_2(x, t), & u_1(x, t) < v < u_2(x, t) \\ 0, & v > u_2(x, t) \end{cases} . \tag{23}$$

In the areas of $(x,t)$-plane where one plateau state is realized we have

$$f_s(v,x,t) = \begin{cases} p(x,t), & v < u_2(x,t) \\ 0, & v > u_2(x,t) \end{cases} , \tag{24}$$

and

$$W_s(v,x,t) = \begin{cases} W_1(x,t), & v < u_1(x,t) \\ W_2(x,t), & u_1(x,t) < v < u_2(x,t) \\ 0, & v > u_2(x,t) \end{cases} . \tag{25}$$

The gas-dynamic equations for plateau heights and maximum velocities can be obtained in the same way as for one beam. The equations for maximum velocities are

$$\frac{\partial u_1}{\partial t} + u_1 \frac{\partial u_1}{\partial x} = 0 \tag{26}$$

$$\frac{\partial u_2}{\partial t} + \frac{(u_1 + u_2)}{2} \frac{\partial u_2}{\partial x} = 0 , \tag{27}$$

and continuity equations

$$\frac{\partial p_1}{\partial t} + \frac{u_1}{2} \frac{\partial p_1}{\partial x} = 0 , \tag{28}$$

$$\frac{\partial p_2}{\partial t} + \frac{(u_1 + u_2)}{2} \frac{\partial p_2}{\partial x} = 0 . \tag{29}$$

On the base of equation (8) we find equations for the spectral energy densities of plasma oscillations taking into account (Eq.(22)) and (Eq.(23))

$$\frac{\omega_p}{m} \frac{\partial}{\partial v} \frac{1}{v^3} \frac{\partial W_1}{\partial t} = \frac{\partial p_1}{\partial t} + v \frac{\partial p_1}{\partial x}, v < u_1(x,t) , \tag{30}$$

$$\frac{\omega_p}{m} \frac{\partial}{\partial v} \frac{1}{v^3} \frac{\partial W_2}{\partial t} = \frac{\partial p_2}{\partial t} + v \frac{\partial p_2}{\partial x}, u_1(x,t) < v < u_2(x,t) , \tag{31}$$

and boundary conditions for the spectral energy density

$$(W_2 - W_1) \frac{\partial u_1(x,t)}{\partial t} = 0, v = u_1(x,t) , \tag{32}$$

$$\frac{\partial W_2}{\partial t} = \frac{\partial W_1}{\partial t} = 0, v = u_1(x,t) , \tag{33}$$

$$W_2 \frac{\partial u_2(x,t)}{\partial t} = 0, v = u_2(x,t) , \tag{34}$$

$$\frac{\partial W_2}{\partial t} = 0, v = u_2(x,t) . \tag{35}$$

In case the electron distribution function consits of only one plateau, gas-dynamic equations are the same as equations (6), (7) and (9) to (11), but maximum velocity is $u_2(x,t)$ instead of $u(x,t)$ and boundary conditions should be completed with equations (32) and (33). The dynamics of the two beam flying-off is described by the equations (26)-(35) when $p_1(x,t) > p_2(x,t)$ and equations (6), (7), (9)-(11), (32)-(33) with changes mentioned above.

## 5 Solution of gas-dynamic equations for a two beam source

As in the case of one beam, the spectral energy density of plasma oscillations differs from zero after the steady state of quasi-linear relaxation has been formed for the quasi-linear time . Then equations (32), (34) with (26), (27) give us

$$u_1(x,t) = const = u_{10},\\ u_2(x,t) = const = u_{20}\ . \tag{36}$$

Using equation (17) the solutions for plateau heights are obvious from equations (28), (29) and (7), where we have changed from $u(x,t)$ to $u_2(x,t)$

$$p_1(x,t) = \frac{n_1}{u_1} exp(-|x - u_1 t/2|/d)\ , \tag{37}$$

$$p_2(x,t) = \frac{n_1}{u_2 - u_1} exp(-|x - (u_1 + u_2)t/2|/d)\ , \tag{38}$$

$$p(x,t) = \frac{n_1 + n_2}{u_2} exp(-|x - u_2 t/2|/d)\ , \tag{39}$$

(here and further index "0" for $u_{10}$, $u_{20}$ will be omitted). The solution of equations (30), (31) for $W_1$ and $W_2$ can be found involving initial conditions at $t = 0$. The analysis of the results thus obtained shows that solution depends upon the correlation between beam parameters. There are three cases available, i.e., when the slow beam has

a) high density

$$\frac{n_1}{u_1} > \frac{n_2}{u_2 - u_1} \tag{40}$$

b) moderate density

$$\frac{n_1}{u_1} < \frac{n_2}{u_2 - u_1}, \frac{n_1}{u_1^2} < \frac{n_2}{u_2^2 - u_1^2} \tag{41}$$

c) low density

$$\frac{n_1}{u_1^2} > \frac{n_2}{u_2^2 - u_1^2}\ . \tag{42}$$

Let us discuss every case separately.

a) The number of slow particles is enough to form steady state (Eq.(22), Eq.(23)) at $t = 0$. Then we have for $p_1(x,t)$ and $p_2(x,t)$ (Figure 3a)

$$p_1(x,t) = \frac{n_1}{u_1} \begin{cases} exp(-|x - u_1 t/2|/d), & x,t \in B_1 \cup C \cup B_3 \\ exp(-\frac{u_2 + u_1}{u_2 - u_1}|x - u_1 t/2|/d), & x,t \in B_2 \end{cases} \tag{43}$$

$$p_2(x,t) = \frac{n_2}{u_2 - u_1} exp(-|x - (u_2 + u_1)t/2|/d)\,,\ x,t \in C \cup B_1 \cup B_2 \cup B_3\ , \tag{44}$$

and for $W_1(v,x,t)$ and $W_2(v,x,t)$

$$W_1(v,x,t) = \frac{mv^4}{\omega_p u_1}((u_1 - v)p_1(x,t) + vp_1(x,t=0))\ , \tag{45}$$

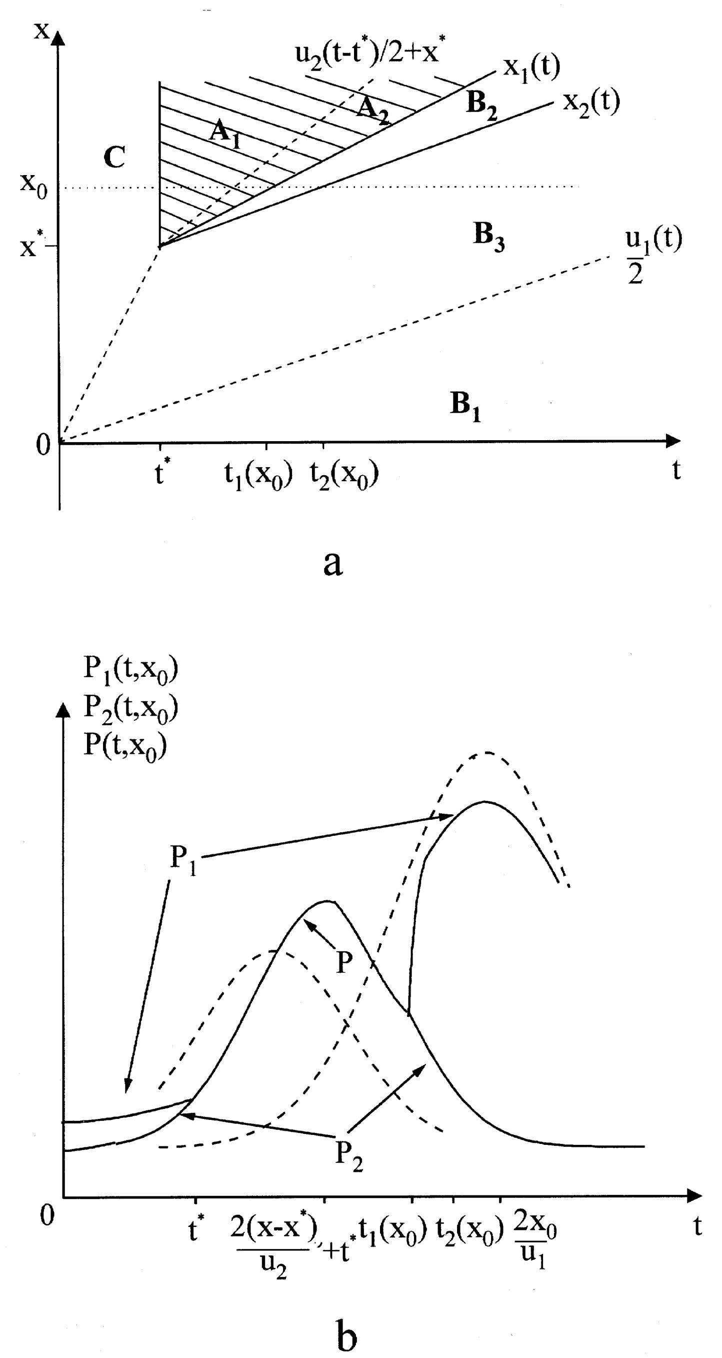



*Figure 3: (a) Areas in $(x, t)$-plane corresponding to the different solutions in the case of a slow beam of low density. (b) Plateau height changing with time at the $x = x_0$ in the case of slow beam of low density. Beam-plasma structures propagating independently are shown by the dashed lines.*

$$W_2(v,x,t) = \frac{mv^3(v-u_1)}{\omega_p(u_1+u_2)}((u_2-v)p_2(x,t) + (v+u_1)p_2(x,t=0)) \ . \qquad (46)$$

Areas $A - C$ in $(x,t)$-plane (Figure 3a) are determined as: $C = \{\frac{u_2+u_1}{2}t < x, t < t^*\}$, $A_1 = \{x > \frac{u_2}{2}(t-t^*) + x^*, t > t^*\}$, $A_2 = \{x_1(t) < x < \frac{u_2}{2}(t-t^*) + x^*\}$, $B_2 = \{x_2(t) < x < x_1(t)\}$, $B_3 = \{\frac{u_1 t}{2} < x < x_2(t), x < \frac{u_1+u_2}{2}t\}$, $B_1 = \{x < u_1 t/2, t > 0\}$,

where

$$x_1(t) = \frac{(u_2+u_1)}{4}(t-t^*) + x^*, x^* = \frac{u_1+u_2}{2}t^*, t^* = \frac{2d}{u_2}ln\left(\frac{n_1(u_2-u_1)}{n_2 u_1}\right) \ , \qquad (47)$$

$$x_2(t) = \frac{u_1}{2}(t-t^*) + x^* \ . \qquad (48)$$

At the line $t = t^*$ at $x > x^*$ $p_2(x,t)$ becomes equal to $p_1(x,t)$ and further they evolve together as $p(x,t)$. The only plateau state is realized in area between straight lines $x_1(t)$ and $x_2(t)$. Here

$$p(x,t) = \frac{n_2}{u_2-u_1}\begin{cases} exp(-|x-x^*-u_2(t-t^*)/2|/d), & x,t \in A_1 \\ exp(-\frac{u_2+u_1}{u_2-u_1}|x-x^*-u_2(t-t^*)/2|/d), & x,t \in A_2 \end{cases} \qquad (49)$$

$$W_1(v,x,t) = \frac{mv^4}{\omega_p u_1 u_2}(u_1(u_2-v)p(x,t) + v[(u_1-u_2)p(x,t^*) + u_2 p_1(x,t=0)]) \ . \qquad (50)$$

$$W_2(v,x,t) = \frac{mv^3}{\omega_p u_2}(v(u_2-v)p(x,t) + \frac{1}{u_1+u_2}[u_1(v^2-u_2^2)p(x,t^*) + u_2(v^2-u_1^2)p_2(x,t=0)]) \qquad (51)$$

and $W_2 \neq 0$ at $v = u_1$. The plateau height $p(x,t)$ reaches its maximum value at the straight line $x = \frac{u_2}{2}(t-t^*) + x^*$ after that it reduces and plateau decays into two plateaus - $p_1(x,t)$ and $p_2(x,t)$ which are given by expressions (50), (51). At the line $x_2(t)$ the spectral energy density $W_2(v,x,t)$ equals to zero at $v = u_1$. We see (Figure 3b) two beam-plasma structures are formed and move with constant velocities $u_1/2$ and $u_2/2$. At the distance $x < x^*$ from the source, areas with one plateau are not form. The fast structure moves with speed $(u_1+u_2)/2$ - an average velocity of the electrons located within $u_1 < v < u_2$, the slow structure with velocity $u_1/2$.

b) At $t = 0$ the number of electrons of the slow beam is not enough to form steady state with two plateaus. There is only one plateau $p(x,t)$ at $t = 0$:

$$p(x,t) = \frac{n_1+n_2}{u_2}\begin{cases} exp(-|x-u_2(t-t^{**})/2|/d), & x,t \in A_1' \\ exp(-\frac{u_2+u_1}{u_2-u_1}|x-u_2(t-t^{**})/2|/d), & x,t \in A_2' \end{cases} \ , \qquad (52)$$

and the spectral energy density

$$W_1(v,x,t) = \frac{mv^4}{\omega_p u_2}((u_2 - v)p(x,t) + vp(x,t=0)) \ , \tag{53}$$

$$W_2(v,x,t) = \frac{mv^4}{\omega_p u_2}((u_2 - v)p(x,t) + (v - \frac{n_1 u_2^2}{(n_1+n_2)v})p(x,t=0)) \ , \tag{54}$$

which has a drop at $v = u_1$. In this case areas of $(x,t)$-plane $A' - B'$ defines as $A'_1 = \{x > x'_2(t), t > t^*, x < 0, 0 < t < t^*\}$, $A'_2 = \{x'_1(t) < x < x'_2(t)\}$, $B'_1 = \{x < \frac{u_1}{2}(t - t^{**}), t > t^*\}$, $B'_2 = \{\frac{u_1}{2}(t - t^*) < x < x'_1(t)\}$, where

$$x'_2(t) = (u_2 + u_1)(t - t^{**})/4, \ t^{**} = \frac{2d}{u_2} ln(\frac{u_1(u_2 - u_1)(n_1+n_2)}{n_1 u_2^2 - (n_1+n_2)u_1^2}) \tag{55}$$

$$x'_1(t) = \frac{u_2}{2}(t - t^{**}) \tag{56}$$

Thus we again have a beam-plasma structure moving with velocity $u_2/2$. At the straight line $x'_1(t)$ $W_2$ equals to zero at $v = u_1$ and plateau $p(x,t)$ decays into the two plateaus $p_1(x,t)$ and $p_2(x,t)$ (Figure 4).

$$p_1(x,t) = \frac{n_1+n_2}{u_2} \begin{cases} exp(-|x - u_2(t - t^{**})/2|/d), & x,t \in B'_1 \\ exp(-\frac{u_2+u_1}{u_2-u_1}|x - u_2(t - t^{**})/2|/d), & x,t \in B'_2 \end{cases} \tag{57}$$

grows up reaching the maximum value at $x = \frac{u_1}{2}(t - t^{**})$ and afterwards declines to give one more structure (Figure 4b). Its velocity is $u_1/2$ and its phase - $t^{**}$. It means that slow structure is formed later than the fast one. The spectral energy density $W_1(v,x,t)$ is

$$\begin{aligned} W_1(v,x,t) = \frac{mv^4}{\omega_p u_1 u_2}&(u_2(u_1 - v)p_1(x,t) + \\ &v[p(x,t=0) + (u_2 - u_1)p_1(x, t'_1(x))]) \ . \end{aligned} \tag{58}$$

The plateau height $p_2(x,t)$ and the spectral energy density $W_2(v,x,t)$ in the area $0 < x < x'_1(t)$ change independently on $p_1(x,t)$ and $W_1(v,x,t)$ in accordance with expressions

$$p_2(x,t) = \frac{n_1+n_2}{u_2} exp(-|x - (u_2+u_1)(t - t^*)|/2), \ x,t \in B'_1 \cup B'_2 \tag{59}$$

$$\begin{aligned} W_2(v,x,t) = \frac{mv^3(v-u_1)}{\omega_p u_2(u_1+u_2)}&(vu_2(u_2 - v)p_2(x,t) + \\ (v+u_1)[(u_1+u_2)p(x,t=0) &+ u_1 p_2(x, t'_1(x))]) \ . \end{aligned} \tag{60}$$

c) Since the number of particles in a beam is small in value, we have only one plateau steady state $p(x,t)$ everywhere in $(x,t)$-plane (Figure 5):

$$p(x,t) = \frac{n_1+n_2}{u_2} exp(-|x - u_2 t/2|/d) \tag{61}$$

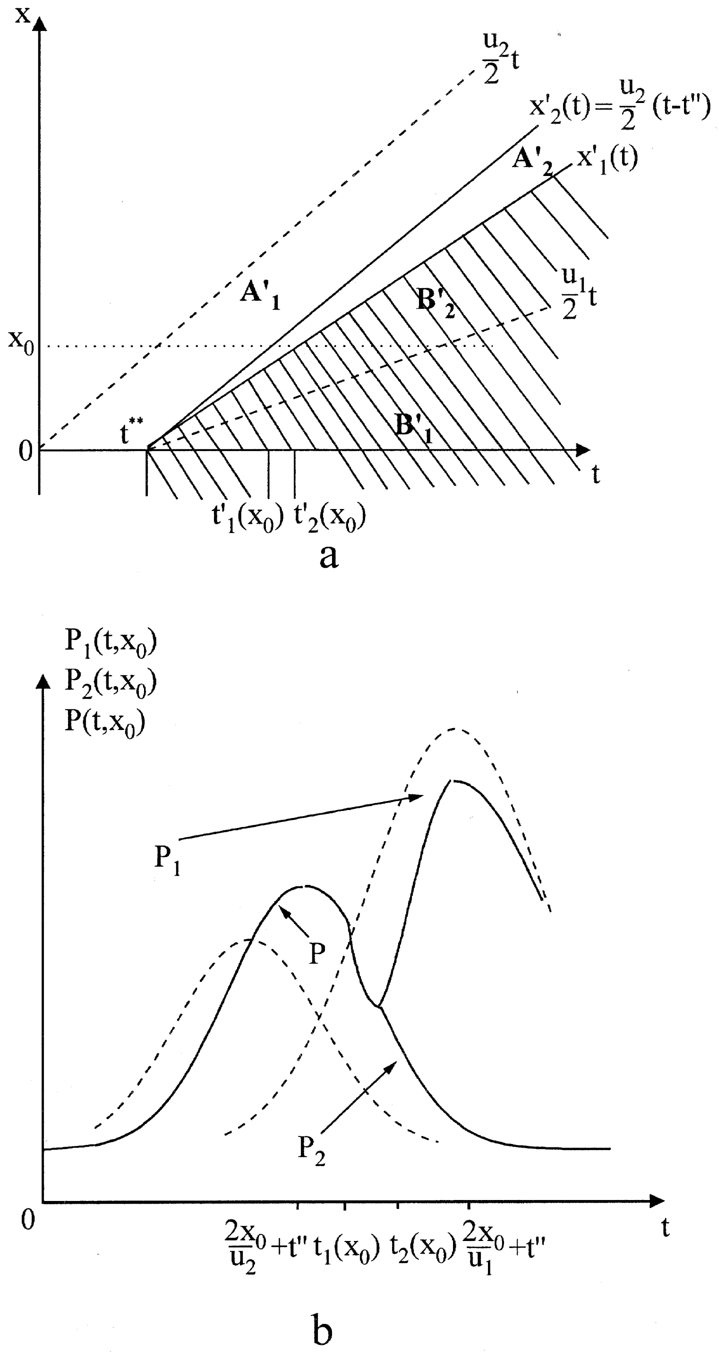
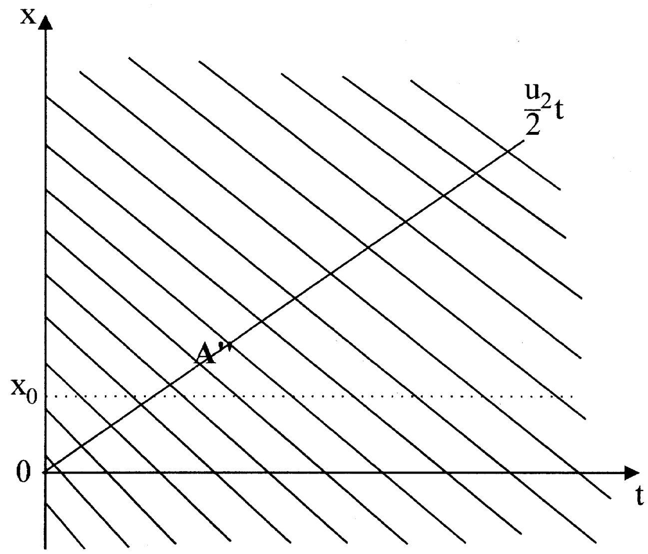
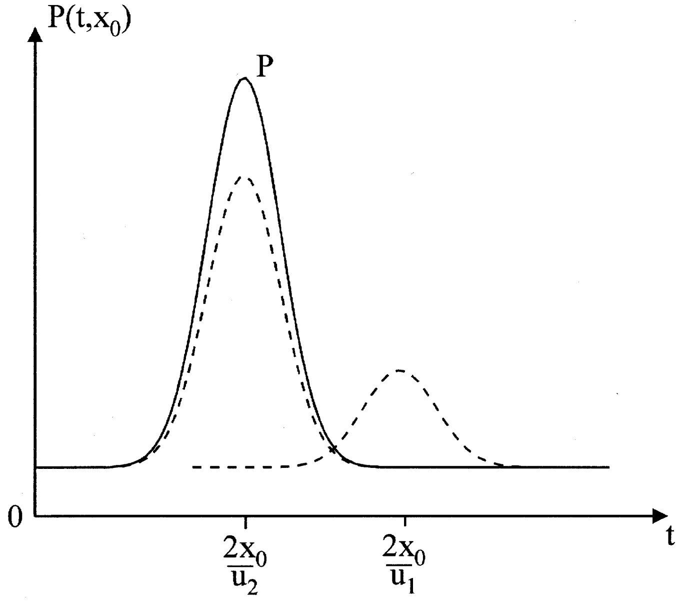



*Figure 4: (a) Areas in $(x, t)$-plane corresponding to the different solutions in the case of slow beam of moderate density. (b) Plateau height changing with time at the $x = x_0$ in the case of slow beam of moderate density. Beam-plasma structures propagating independently are shown by dashed lines.*




a




b

*Figure 5: (a) Areas in $(x, t)$-plane corresponding to the different solutions in the case of slow beam of high density. (b)Plateau height changing with time at the $x = x_0$ in the case of slow beam of high density. Beam-plasma structures propagating independently are shown by dashed lines.*

and the spectral density

$$W_1(v,x,t)=\frac{mv^4}{\omega_p u_2}((u_2-v)p(x,t)+vp(x,t=0)) \tag{62}$$

$$W_2(v,x,t)=\frac{mv^3}{\omega_p u_2}(v(u_2-v)p(x,t)+(v^2-\frac{u_2^2 n_1}{n_1+n_2})p(x,t=0))\ . \tag{63}$$

Despite of the fact that the spectral energy density has a drop at $v=u_1$, $W_2$ at $v=u_1$ differs from zero for all $x,t$ and plateau $p(x,t)$ does not decay into $p_1(x,t)$ and $p_2(x,t)$ like it takes place in cases a) and b). So we have only one beam-plasma structure which moves with velocity $u_2/2$ (Figure 5).

Beam-plasma structures independently on initial conditions are formed only in area $x>0$. Their shapes are determined by the initial spatial distribution of electrons at $t=0$.

## 6 The equations and solution for a set of mono-energetic beams

The initial condition at $t=0$ is a row of mono-energetic beams in velocity space. We also assume that all particles are concentrated near $x=0$

$$f(v,x,t=0)=exp(-|x|/d)\sum_{i=1}^{N}n_i\delta(v-u_i)\ , \tag{64}$$

where $\Delta u=u_i-u_{i-1}$ chosen equal for all $i$ and $n_i>n_{i-1}$ that implies number density declining with velocity. For $\Delta u\rightarrow 0$ the row of beams (Eq.(64)) is implied to be a continuous function of velocity $\sum_{i=1}^{N}n_i\delta(v-u_i)\rightarrow f_0(v)$ with maximum velocity $v=u_{max}=u_N$. Having relaxed the electron distribution function looks like a staircase where the height of the individual steps (plateau heights) decreases with velocity.

$$f_s(v,x,t)=\begin{cases} p_i(x,t), & u_{i-1}(x,t)<v<u_i(x,t) \\ 0, & v>u_N(x,t) \end{cases}\ , \tag{65}$$

$$W_s(v,x,t)=\begin{cases} W_i(v,x,t), & u_{i-1}<v<u_i(x,t) \\ 0, & v>u_N(x,t) \end{cases}\ . \tag{66}$$

Here integrating equations (2) and (3) we obtain gas-dynamic equations for every plateau height and the corresponding spectral density of plasma oscillations. The spectral density $W_i$ is equal to zero at $v=u_{i-1}$ for $t=0$. Thus as in case (a) of two mono-energetic beams (Eq.(37)) we have an equation for each plateau boundary velocity

$$\frac{\partial u_i}{\partial t}+u_i\frac{\partial u_i}{\partial x}=0\ , \tag{67}$$

and continuity equations

$$\frac{\partial p_i}{\partial t}+\frac{(u_i+u_{i-1})}{2}\frac{\partial p_i}{\partial x}=0\ , \tag{68}$$

and for the spectral density of plasma oscillations

$$\frac{\omega_p}{m}\frac{\partial}{\partial v}\frac{1}{v^3}\frac{\partial W_i}{\partial t} = \frac{\partial p_i}{\partial t} + v\frac{\partial p_i}{\partial x}, u_{i-1}(x,t) < v < u_i(x,t) \ , \tag{69}$$

and boundary conditions

$$(W_i - W_{i-1})\frac{\partial u_{i-1}(x,t)}{\partial t} = 0, v = u_{i-1}(x,t) \ , \tag{70}$$

$$\frac{\partial W_i}{\partial t} = \frac{\partial W_{i-1}}{\partial t} = 0, v = u_{i-1}(x,t) \ . \tag{71}$$

Two or more plateaus united form new plateau $p_i^0(x,t)$ with zero minimum velocity. Thus for $j$ steps the equations are the same changing index $i-1$ to 0 and $p_i^0(x,t)$ continues from $v=0$ to $v=u_i(x,t)$. Solving the obtained equations the solutions for plateau heights are

$$p_i(x,t) = \frac{n_i}{u_i - u_{i-1}} exp(-|x-(u_i+u_{i-1})t/2|/d) \tag{72}$$

$$p_i^0(x,t) = \frac{n_i}{u_i - u_{i-1}} exp(-|x-u_it/2|/d) \ . \tag{73}$$

In general the solution yields a row of beam-plasma structures moving with constant velocities $(u_i+u_{i+1})/2$ and $u_i/2$. "Interaction" between them means the transformation of two successive plateaus into one plateau whereby the speed of corresponding beam-plasma structure is decreased. In this paper we consider an electron distribution function $f_0(v)$ where the transformation of plateaus starts at the smallest velocities. The less the speed of the beam-plasma structure is the faster it unites into common a plateau (Figure 6). The two nearest plateaus $p_{i+1}(x,t)$ and $p_i^0(x,t)$ transform into $p_{i+1}^0(x,t)$ at the moment

$$t_i = \frac{2d}{u_iu_{i+1}}\sum_{j=1}^{i} u_j\Lambda_j,$$
$$\Lambda_j = ln\left(\frac{n_j(u_{j+1}-u_j)}{n_{j+1}(u_j-u_{j-1})}\right), \tag{74}$$

when plateau $p_i(x,t)$ reaches $p_i^0(x,t)$ and they unite in the common plateau $p_{i+1}^0(x,t)$. The plateau height of the common plateau $p_{i+1}^0(x,t)$ continues to grow. At the moment $t_{i+1}$ the $i+1$ plateau is involved into the common plateau. Having reached the maximum value the common plateau $p_i^0(x,t)$ begins to decrease and at the straight lines

$$x_i'(t) = \frac{u_{i+1}+u_i}{4}(t-t_i') + x_i',$$
$$x_i' = \frac{u_{i+1}+u_i}{2}t_i' \tag{75}$$

the plateau $p_{i+1}^0(x,t)$ decays. At the straight line $x_i'(t)$ the common plateau has minimum value and the spectral density of plasma oscillation $W_i(v=u_{i-1})=0$. It is a start point where plateau $p_i(x,t)$ separates from the common plateau. Further the evolution of each plateau goes in different way.

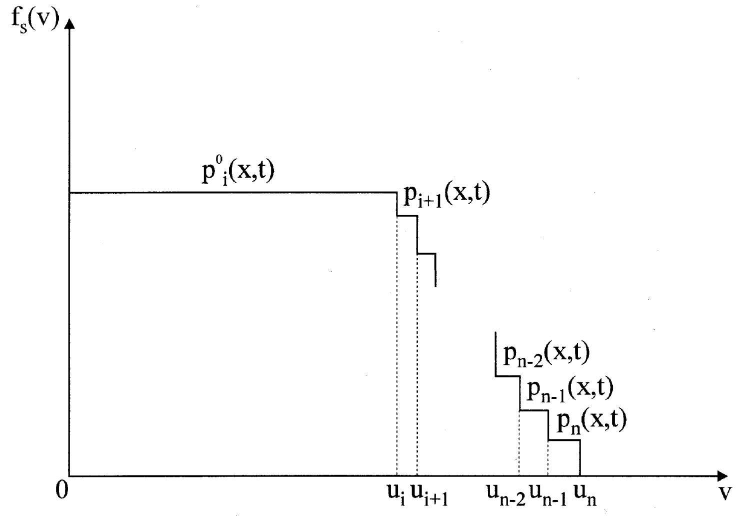



*Figure 6: Steady state: electron distribution function for N mono-energetic beams.*

Thus the evolution of the common plateau is the continuity of ups and downs. Increasing the number of electron beams, $N$, the width of beam plasma structure is narrowed in space in accordance with

$$d_i^+ \approx d_i^- \approx \frac{d}{2i} \ . \tag{76}$$

However, the particle number in the structure retains defined by the initial number density.

The absorption of plasma waves accelerates electrons and forces them to concentrate near the average velocity of the plateau which in turn decreases the number of electrons which have velocity less than the average one. Being interested in general properties of electron flying-off let $N$ be a big number and let us turn to the average electron distribution function (over time $\Delta t \gg d/Nv$). This electron distribution function appeares to be a plateau growing with time

$$< p_i^0(x,t) >= f_0(\frac{2x}{t-t_i})\frac{d}{x-x_i} \ , \tag{77}$$

and with decreasing boundary velocity

$$u_{max}(x,t) = \frac{2x}{t-t_i} \ . \tag{78}$$

This average values behave similar to the Rutov and Sagdeev asymptotic solution [Rutov and Sagdeev, 1970].

## 7 Conclusion

Electron beams propagating through plasmas generate Langmuir waves which, in turn, have an influence on electron distribution function through the acceleration (for $v < u/2$) or deceleration of (for $v > u/2$) particles. At the beginning faster electrons create plasma osillations at levels differing from zero. Later slower particles decrease the number of plasmons. Coming into a new point fast electrons emit plasma waves and that process goes on and on in every point where the first particles arrive. Despite the fact that Langmuir waves are considered to have zero group velocity the spectral energy of plasma waves is changed in accordance with electron movement in space. Such emission and absorption of plasmons by electrons force the electron beam to propagate in plasma as a beam-plasma structure. Both plateau height and spectral energy density of plasma oscillations have spatial maximum at straight line $x = ut/2$ and this maximum moves with constant velocity $u/2$.

As two mono-energetic beams $f_1 = n_1\delta(v - u_{10})$, $f_2 = n_2\delta(v - u_{20})$ are propagating through the plasma, electrons of the slow beam are accelerated by plasma waves of the fast structure. These electrons overtake then the fast structure leading to an "interaction" process. To understand the features of this "interaction" we compare the two-beam plasma structure propagation with the propagation of individual structures. In the independent propagation case each structure moves with constant velocity which equals half the maximum mono-energetic velocity, i.e., the one we obtain in the case of a single mono-energetic beam. When slow beam has high density (see Eq.(40)), there are two beam plasma structures near the injection point. The fast structure moves with speed $\frac{u_1 + u_2}{2}$ and slow structure velocity is $\frac{u_1}{2}$. At that point a part of slow structure particles goes into the fast one, and their shapes are delayed to be pulled in each other direction. Starting from $x^*$ fast structure velocity becomes less and equal to $u_2/2$, and its phase is that more early injection time is seemed (Figure 3a). The fast structure delaying arises after the plateau $p_2(x,t)$ becomes equal $p_1(x,t)$ they start to evolve together. The average speed is decreased that delays the new structure. For moderate density slow beam (Eq.(41)) we have also two beam-plasma structures. However, slow structure phase is different from zero - slow structure is formed later than fast one. Again a part of electrons of slow beam-plasma structure is accelerated into fast structure and area between structures is impoverished with particles.

In case of slow number density of slow beam (Eq.(42)), the slow beam-plasma structure is not formed - it is completely absorbed by fast structure whose velocity is $u_2/2$, its shape is symmetrical and only its amplitude is renormalized (Figures 5a, b).

In case there are initially $N$ mono-energetic beams the electron distribution function in every point is determined by the "interaction" of beam-plasma structure. It looks like a common plateau and a staircase. These stairs grow with time and attache to the common plateau. When the spectral energy density of plasma waves turns to be equal to zero at $v = u_i$ the respective stair ($u_i < v < u_{i+1}$) is separated from the common plateau. Thus the maximal velocity of the common plateau is decreased jumping down from $v = u_{i+1}$ to $v = u_i$ but its height is increased. For velocities $v$ which are greater than the maximal velocity $u_i$ the electron distribution function has a form of staircase with decreasing stair heights with velocity (Figure 6). The average plateau height $< p_i^0(x,t) >$ increases with time but the boundary velocity of the plateau decreases.