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Non-local time-dependent treatments of convection in A-G type stars

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Abstract

Time-Dependent Convection (TDC) models obtained by combining the local treatment of Gabriel (1996) and Grigahcène et al. (2005) and the non-local prescriptions of Spiegel (1963) are presented. We show that in the stationary unperturbed case, these non-local treatments can be constrained by the results of 3D hydrodynamic simulations (Stein & Nordlund 1998). We consider here the case of solar-type stars with a large convective envelope and A-F type stars with two very thin surface convection zones.

1. Local treatments

The first treatments proposed for the modeling of convection in stellar interiors were local. In these models, the convective flux, Reynolds stress, ... are directly related to mean local thermodynamic quantities such as the temperature, the entropy, the density and their gradients. This approximation is justified when the space scale associated to most energetic turbulent motions is small compared to the scale height of the mean quantities (e.g. $H_{\rm p} = |{\rm d}\ln P/{\rm d}r|^{-1}$). Unfortunately, it is seldom the case in stellar interiors. In the local approach, the correlation terms associated to turbulence are obtained at each given radius by solving the equations for the convective fluctuations, assuming constant coefficients. However, there appear more unknowns than equations in such approach and some assumptions must be done for the closing of the system. In the simplest case called the Mixing-Length Theory (MLT, Böhm-Vitense 1958), this closure is obtained by reducing the turbulent spectrum to its simplest expression: a scalar l related to some characteristic scale of the stratification; most frequently it is assumed $l = \alpha|{\rm d}\ln P/{\rm d}r|^{-1}$, with the free Mixing-Length (ML) parameter α. More sophisticated local theories have been proposed taking the full spectrum of turbulence into account (Canuto & Mazzitelli 1991). These local treatments are easily implemented in a stellar evolution code and the free parameters (e.g. α) can be calibrated from the Solar age, radius and effective temperature. For this reason, they have been and are still widely used.

If we consider now the interaction between convection and pulsations, the things are much more complex. In the framework of the MLT, different approaches leading to the same equations in the stationary case differ significantly for the time-dependent perturbed case: on one hand we find the TDC theory of Gough (1977) and on the other hand the TDC theory of Gabriel (1996). The unphysical short-wavelength spatial oscillations of the eigenfunctions found with these treatments in cool stars ($T_{\rm eff} \lesssim 6500$ K) was considered for a long time as the main criticism of the local approach, and it was pointed out that non-local treatments could solve this problem (Gonczi 1986). However, Grigahcène et al. (2005) proposed recently a new perturbation of the closure equations that allows to solve this problem in a local way (see also Dupret et al., these proceedings).

We note that some good results can already be obtained with local treatments such as the stabilization of the modes at the red side of the δ Sct instability strip, the explanation of the driving of the γ Dor g-modes (Dupret et al. 2005a) and better multi-color photometric amplitude ratios and phase differences (Dupret et al. 2005b, 2005c).

2. Non-local treatments

In most of the cases, the space scale of most energetic turbulent motions is larger than the scale height of the mean stratification in stellar interiors, so that the local treatments are not justified. The only way to model rigorously the non-local character of convection is through 3D hydrodynamic simulations. However, such computations cost a lot of time and they cannot be used to determine the coherent interaction between convection and pulsations. For these reasons, it is useful to derive non-local treatments based on simpler approaches such as the MLT. The original idea of such treatment comes from Spiegel (1963) and is based on an analogy with the radiative transfer in stellar atmospheres. The well known transfer equation is:

$$\mu \mathrm{d}I_\nu/\mathrm{d}\tau_\nu = S_\nu - I_\nu \,, \tag{1}$$

where $\mathrm{d}r/\mathrm{d}\tau_\nu = (\kappa_\nu \rho)^{-1}$ is the mean free-path of the photons, and S_ν is the source given by the Planck function in the Local Thermodynamic Equilibrium (LTE) approximation. Similarly, in a very simplified way, we write as follows the motion and energy equations for a convective element:

$$\mathrm{d}V/\mathrm{d}t = -g\,\Delta\rho/\rho - V^2/l \,, \tag{2}$$
$$\mathrm{d}\Delta s/\mathrm{d}t = -V\mathrm{d}s/\mathrm{d}r - \omega_{\mathrm{R}}\Delta s - V\Delta s/l \,, \tag{3}$$

where l is the mean free-path of the convective element and ω_{R} corresponds to the radiative losses. In the local MLT approach, we assume that $|\mathrm{d}V/\mathrm{d}r| \ll |V|/l$ (idem for Δs). In the stationary case and defining $\omega_{\mathrm{c}} = V/l$ (ω_{c}^{-1} is the life-time of the convective elements), we find the simple eigenvalue problem:

$$\omega_{\mathrm{c}} V = -g\,\Delta\rho/\rho \,, \tag{4}$$
$$\omega_{\mathrm{c}}\Delta s = -V\mathrm{d}s/\mathrm{d}r - \omega_{\mathrm{R}}\Delta s \,. \tag{5}$$

The characteristic polynomial of this problem has a positive and a negative eigenvalue inside a convective zone. The positive eigenvalue and the corresponding eigenvector give the classical local MLT solution.

We come back now to the "real" non-local problem. If we assume that (V, Δs) is in the eigenspace of the local MLT solution, then we can write in the stationary case:

$$\mathrm{d}V/\mathrm{d}\zeta = l\omega_{\mathrm{c}} - V \,, \tag{6}$$
$$\mathrm{d}\Delta s/\mathrm{d}\zeta = l\omega_{\mathrm{c}}\Delta s/V - \Delta s \,, \tag{7}$$

where $\mathrm{d}\zeta = \mathrm{d}r/l$ and $l\omega_{\mathrm{c}}$ is the velocity corresponding to the local solution: V_{loc}. The form of this system is similar to the transfer equation (1) with a source function given by the local MLT solution, and ζ which can be compared to the optical depth. We consider now only the first of these two equations. Integrating it for convective elements going up, we have: $V^{+}_{\mathrm{non-loc}}(\zeta_0) = \int_{-\infty}^{\zeta_0} V_{\mathrm{loc}} \exp(\zeta - \zeta_0)\mathrm{d}\zeta$. Combining elements going up and down and taking the mean gives:

$$V_{\mathrm{non-loc}}(\zeta_0) = \int_{-\infty}^{+\infty} V_{\mathrm{loc}} e^{-|\zeta-\zeta_0|}\mathrm{d}\zeta. \tag{8}$$

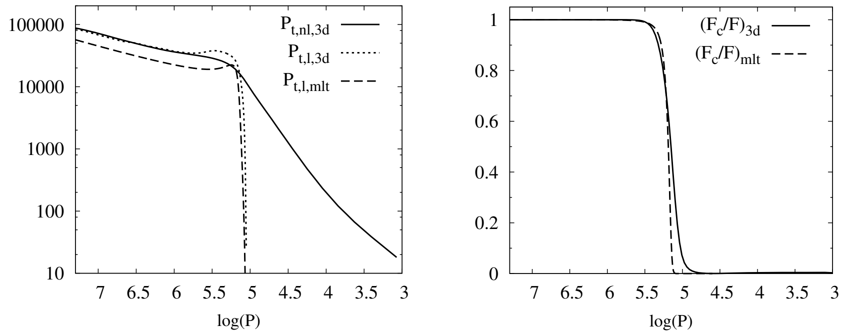

Figure 1: Turbulent pressure (left) and convective flux relative to total flux (right) as obtained with 3D hydrodynamic models (Stein & Nordlund 1998) and MLT for the Sun.

We assume now that such kind of relation can be used to relate any non-local (nl) and local (l) quantity associated to turbulence. This is of course an approximation and it is appropriate to introduce free non-local parameters a and b at this level. Following Balmforth (1992), we get for the turbulent pressure and convective flux:

$$P_{\rm t,nl}(\zeta_0) = \int_{-\infty}^{+\infty} P_{\rm t,l} e^{-b|\zeta-\zeta_0|} {\rm d}\zeta \ , \quad F_{\rm c,nl}(\zeta_0) = \int_{-\infty}^{+\infty} F_{\rm c,l} e^{-a|\zeta-\zeta_0|} {\rm d}\zeta \, . \tag{9}$$

Taking the second order derivative gives the two very simple differential equations:

$${\rm d}^2 P_{\rm t,nl}/{\rm d}\zeta^2 = b^2 (P_{\rm t,nl} - P_{\rm t,l}) \ , \quad {\rm d}^2 F_{\rm c,nl}/{\rm d}\zeta^2 = a^2 (F_{\rm c,nl} - F_{\rm c,l}) \, . \tag{10}$$

The system of equations for the equilibrium stellar models is then entirely defined. In the energy and hydrostatic equilibrium equations appear the non-local convective flux and turbulent pressure respectively. We add the two differential equations (10) relating them to the local ones. Finally, the adopted local convection treatment enables us to relate the local turbulent quantities to the usual variables (T, ρ, $\ldots$) and their gradients. If the MLT is used at this level, we call this treatment the non-local MLT.

The same approach can be followed for the modeling of non-adiabatic pulsations. We add to the equations of the linear theory the perturbations of equations (10). And a local TDC treatment (see previous section) enables us to relate the perturbed local convective flux and turbulent pressures to the usual perturbed mean quantities. This system of equation can be implemented and solved by a non-adiabatic pulsation code, allowing in particular a better determination of the damping rates of the modes in solar-type stars.

3. Constraints from 3D hydrodynamic models

3D hydrodynamic models can be used to test and constrain simpler convection models such as the non-local MLT treatment presented in the previous section. We consider here the case of the Sun and use the results of a 3D simulation by Stein & Nordlund (1998) with a resolution of $125 \times 125 \times 82$ over the span of 1 hr. From the 3D results, the turbulent pressure and convective flux can be determined by taking the appropriate means ($P_{\rm t} = \overline{\rho V_r^2}$, $F_{\rm c} = \overline{\rho V_r \delta h}$). In Fig. 1, we compare the 3D results with those obtained with local MLT treatment. The solid lines are the 3D values of $P_{\rm t}$ (left) and $F_{\rm c}/F$ (right) and the dashed lines the local MLT results. Of particular interest is the behaviour of the turbulent pressure in the overshooting region ($5 \geq \log P \geq 3$), which corresponds essentially to an exponential decrease in the 3D results. The non-local treatment proposed in the previous section (Eq. 9) predicts very similar $P_{\rm t,nl}$ in the overshooting region with a non-local parameter $b \simeq 3$. This indicates that it could be possible to mimic the 3D results with simpler models. The convective flux predicted by 3D and local MLT models are less different (right panel). The values of the non-local parameter a required to fit the 3D results are larger than b, we find $a \simeq 11$. Following the approach of previous section, local and non-local variables are put in relation throughout Eq. (10). If we take the non-local turbulent pressure $P_{\rm t,nl,3d}$ from the 3D simulations, we can deduce the corresponding local one by: $P_{\rm t,l,3d} = P_{\rm t,nl,3d} - (1/9)\,{\rm d}^2(P_{\rm t,nl,3d})/{\rm d}\zeta^2$; this gives the dotted line of Fig. 1. Comparing it with the dashed line (local MLT) shows a similar shape. In the illustrated case, the ML parameter $\alpha = 1.75$ (solar calibrated value). Our most recent investigations show that by choosing appropriately α (letting it vary with depth) and the non-local parameters a et b, it is possible to reproduce very closely the results of the 3D simulations with non-local MLT.

We remark finally that similar approach can be followed for the modeling of A-F stars. These stars have two thin surface convection zones in the partial ionization zones of $\text{He}_{\rm II}$ and H. Works by Kupka & Montgomery (2002) show that these two zones are dynamically connected (the velocity of turbulent elements does not drop to zero between them), but not thermally connected (the fraction of energy transported by convection drops to zero between them). As for the solar case, the required values for the non-local parameter a are larger than b. The stability of the intermediate order p-modes is affected by the details of the physics in this region; it is thus important to model it correctly in the non-adiabatic seismic study of A-F stars.

Further works are to perform non-adiabatic computations with non-local time-dependent convection MLT treatment, using structure models mimicking the results of 3D simulations. This will allow us to better determine the theoretical damping rates for solar-type oscillations. These works are very important for the interpretation of the future observations of the space mission COROT.

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