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The Effect of the Counter-Rotating Terms on the Rabi Oscillations and Steady States in the Few-Atom Jaynes-Cummings Model with Cavity Losses

By

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Abstract

The effect of the counter-rotating terms in the few-atom Jaynes-Cummings model is examined in both lossless cavities and cavities with losses. Numerical results for the time evolutions of the atomic populations inversion and dipole moment for an initial coherent field are presented for various numbers of atoms. As a consequence of the counter-rotating terms, the appearance of new steady states for atomic population inversion in the case of cavities with losses is pointed out.

Key words: Quantum optics, Jaynes–Cummings model, counter–rotating terms, Rabi oscillations.

1 Introduction

The experimental realization of the Jaynes-Cummings model (JCM) in experiments with Rydberg atoms [1] led to intensive theoretical studies of the model in the so-called rotating-wave approximation (RWA) (the neglect of the "energy-non-conserving", counter-rotating terms (CRT).

However, in our previous papers, it has been shown that the CRT can lead to significant changes in the Rabi oscillations and squeezing in the case of the *single-* and *many-atom JCM* in a *cavity without losses* [2, 3]. Analogous investigations in *cavities with losses*, which were restricted to the case of the *single-atom JCM* [4], have confirmed the significance of the contribution of the CRT. The aim of the present paper is to extend the investigations in damped cavities to the few-atom JCM.

Recently, in a cavity with losses the appearance of new steady states for atomic population inversion has been pointed out in the case of a single-atom JCM [4, 5]. In the present paper we shall study the influence of the CRT on the dynamical and steady-state behaviour of the atomic population inversion and dipole moment in the case of the *few-atom* JCM with cavity damping for an initial coherent field.

2 Exact Equations for Density Matrix Elements and Numerical Results

The Liouvillian for the few-atom JCM model without RWA in the interaction picture reads are

$$L(t) = L_{AR}(t) + i\Lambda_R, \quad L_{AR}(t) = [H_{AR}(t), \dots] \tag{1}$$

with corresponding atom-field interaction Hamiltonian ($\hbar = 1$)

$$H_{AR}(t) = g(s^- e^{-i\omega t} + s^+ e^{i\omega t}) \otimes (a e^{-i\omega t} + a^+ e^{i\omega t}), \quad s^\pm = \sum_{i=1}^{N} s_i^\pm \tag{2}$$

and the field-damping Liouvillian

$$\Lambda_R(\dots) = \kappa([a(\dots), a^+] + [a, (\dots)a^+]). \tag{3}$$

Here $s_i^\pm$ are the atomic dipole-moment operators of the ith atom, N is the number of atoms, ω is the frequency of the atomic transition and the frequency of the resonant cavity field mode, a^+ and a are the photon creation and annihilation operators, g is the atom-field coupling constant, and κ is the cavity damping factor.

From the Liouville equation the following equation for the density matrix elements $\rho^*_{n(k),l(m)} = \rho_{l(m),n(k)}$ can be derived:

$$\frac{d\rho_{n(k),l(m)}}{dt} = -ig[\sqrt{(N-n)(n+1)(k+1)}\,\rho_{n+1(k+1),l(m)}$$

$$-\sqrt{(N-l)(l+1)(m+1)}\,\rho_{n(k),l+1(m+1)} + \sqrt{(N-n+1)nk}\,\rho_{n-1(k-1),l(m)}$$

$$-\sqrt{(N-l+1)lm}\,\rho_{n(k),l-1(m-1)} + \sqrt{(N-n+1)n(k+1)}\,e^{-2i\omega t}\rho_{n-1(k+1),l(m)}$$

$$-\sqrt{(N-l+1)\,l(m+1)}\,e^{2i\omega t}\rho_{n(k),\,l-1(m+1)}+\sqrt{(N-n)(n+1)\,k}\,e^{2i\omega t}\rho_{n+1\,(k-1),\,l(m)}$$

$$-\sqrt{(N-l)\,(l+1)\,m}\,e^{-2i\omega t}\rho_{n(k),\,l+1\,(m-1)}]+2\kappa\sqrt{(k+1)(m+1)}\,\rho_{n(k+1),\,l(m+1)}$$
$$-\kappa(k+m)\,\rho_{n(k),l(m)},\, n,l=0,1,\ldots,N,\quad k,m=0,1,2,\ldots \tag{4}$$

where we used the basis vectors

$$|n(k)\rangle \equiv |s,s-n\rangle_A \otimes |k\rangle_R,\, s=\frac{N}{2},\, n=0,1,\ldots,N,\, k=0,1,2,\ldots, \tag{5}$$

with $|s,s-n\rangle_A$ as the Dicke atomic states [6] and $|k\rangle_R$ as the Fock states of the radiation field with k photons.

We solve the above equation for a special initial condition $\rho(0)=\rho_A(0)\otimes\rho_R(0)$ with atoms being in a coherent superposition of the two uppermost states

$$\rho_A(0)=|\psi_A(0)\rangle\langle\psi_A(0)| \tag{6}$$

$$|\psi_A(0)\rangle=\sin\theta|0\rangle+\cos\theta|1\rangle,\quad |n\rangle\equiv|s,s-n\rangle \tag{7}$$

and the radiation field initially in a coherent state

$$|\psi_R(0)\rangle=|\alpha\rangle=\sum_{k=0}^{\infty}\frac{(\alpha)^k}{\sqrt{k!}}\exp\left(-\frac{|\alpha|^2}{2}\right)|k\rangle_R. \tag{8}$$

We now solve the above Eq. (4) numerically within the RWA and without the RWA for different numbers of atoms, $N=1, N=2$ and $N=3$, and $|\alpha|^2=10$ in the case of an undamped and a damped cavity with damping: $K=\kappa/g=2$

In Figs. 1–6 we plot the time evolutions of the atomic population inversion

$$Z(T=gt)=\frac{Tr[s_z\rho(t)]}{N}=\frac{\langle s_z\rangle_t}{N}, \tag{9}$$

the real and imaginary part of the atomic dipole moment

$$RSP(T)=\frac{\mathrm{Re}(\langle s^+\rangle_t)}{N},\quad ISP(T)=\frac{Im(\langle s^+\rangle_t)}{N}, \tag{10}$$

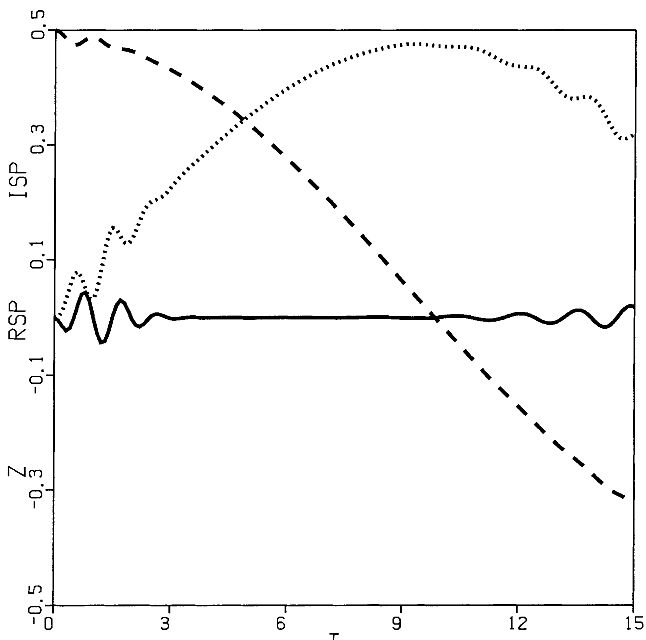
within and without the RWA in the case of both lossless cavities and cavities with losses.

From Figs. 1–3, it can be seen that, as a consequence of the CRT, the Rabi oscillations of the atomic population inversion do not collapse. Their amplitudes decrease with the increasing number N of atoms.

The comparison with our previous results [2], where the phase-diffused coherent field has been investigated, shows that, in the present case, the effect of the CRT becomes much stronger. This increase is a consequence of the nonvanishing nondiagonal field density matrix elements.

In Figs. 4a–6a, the case of stronger cavity damping $K = 2$ is investigated in the RWA. The photons are not stored long enough in the cavity to be reabsorbed, therefore no Rabi oscillations occur. Collective radiation effects cause a more rapid decay of the atomic population inversion.

As Figs. 4b–6b show, the contribution of the CRT changes totally the time behaviour of the decaying system and prevents a total de-excitation



a

Fig. 1. Time evolutions of the atomic population inversion Z (solid line), the real and imaginary part of the atomic dipole moment RSP (dashed line) and ISP (dotted line) as functions of the scaled time $T = gt$ for a lossless cavity ($K = 0$) in the case of an initial coherent superposition of the two uppermost atomic states ($\theta = 0.25\,\pi$), and initially coherent cavity field ($\alpha = \sqrt{10}$). The investigations are carried out in the single-atom case ($N = 1$): **a** within RWA; **b** without RWA for the coupling $W = \omega/g = 2$

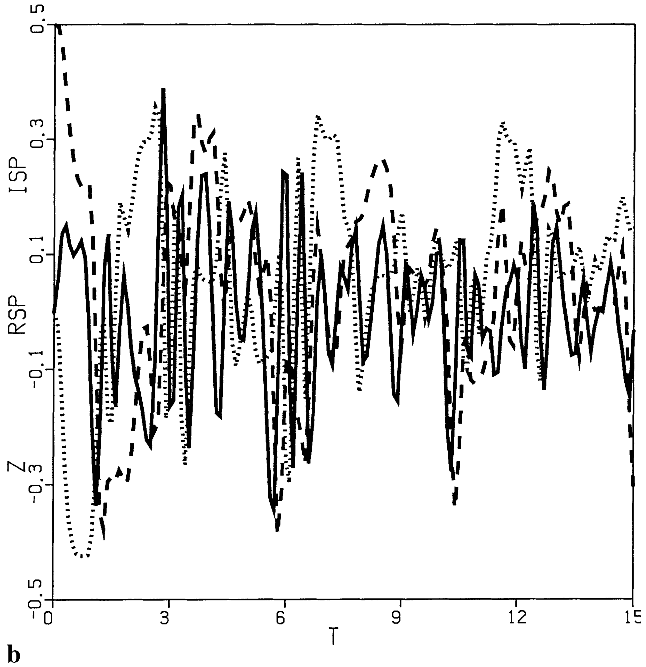


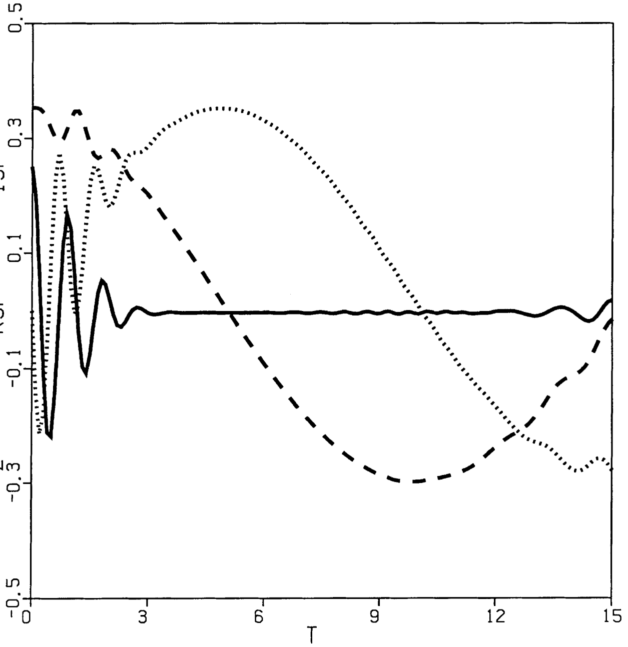
Fig. 1b

($Z = -0.5$) of the atomic system. This leads to the appearance of *new steady states* whose energy values lie appreciably above the ground-state value: $Z = -0.5$.

3 Conclusion

In Section 2, in the case of the few-atom JCM, we derived an exact recurrence equation for density matrix elements without making the RWA. We solved this equation numerically for atoms being initially in a coherent superposition of the two uppermost states and the radiation field in an initial coherent state. Both lossless cavities and cavities with losses were studied.

Numerical results for the time evolutions of the atomic population inversion and dipole moment for an initial coherent field were presented for various numbers of atoms. As a consequence of the CRT, the appearance of new steady states for atomic population inversion in the case of cavities with losses is pointed out.



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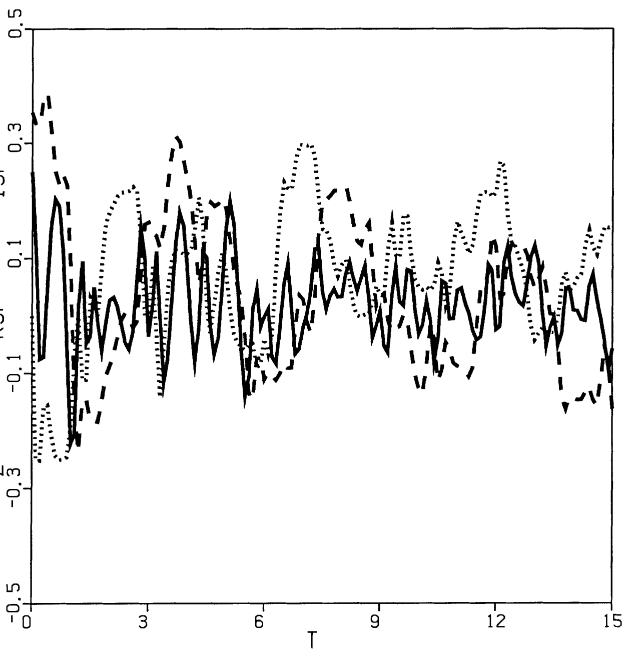
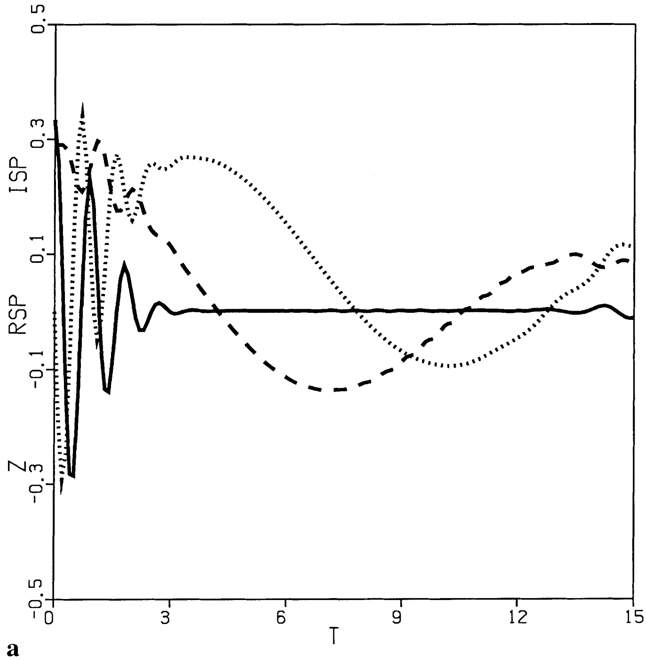


b

Fig. 2. Same as Fig. 1 for $N = 2$



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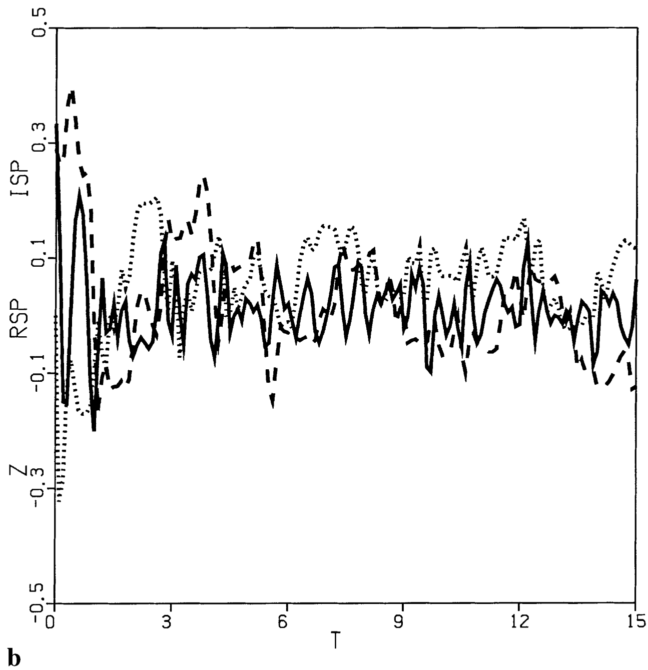
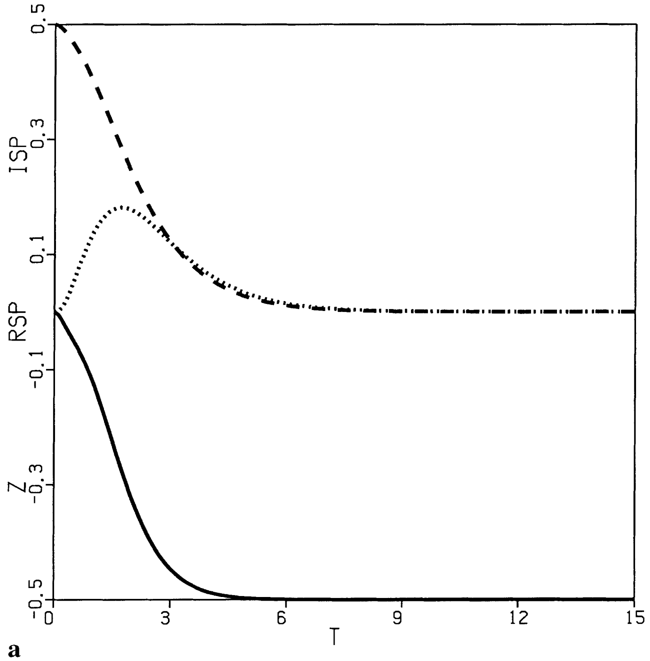


b

Fig. 3. Same as Fig. 1 for $N = 3$



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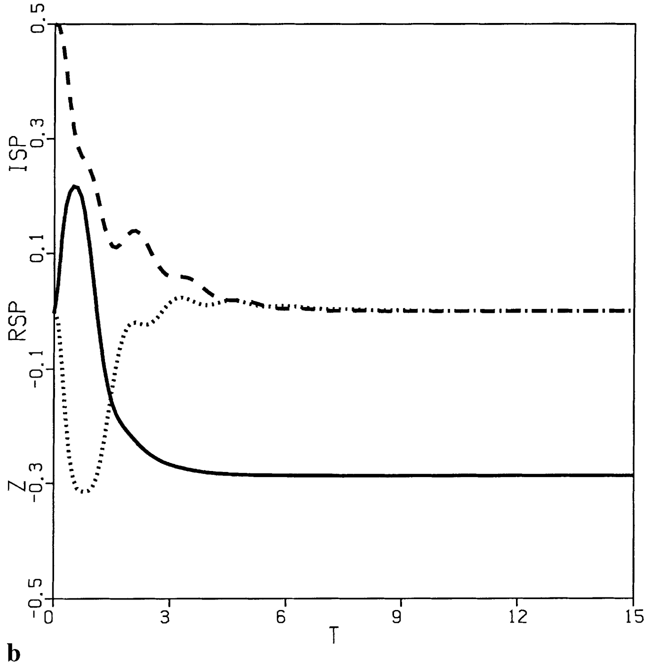

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Fig. 4. Same as Fig. 1 for a damped cavity: $K = 2$

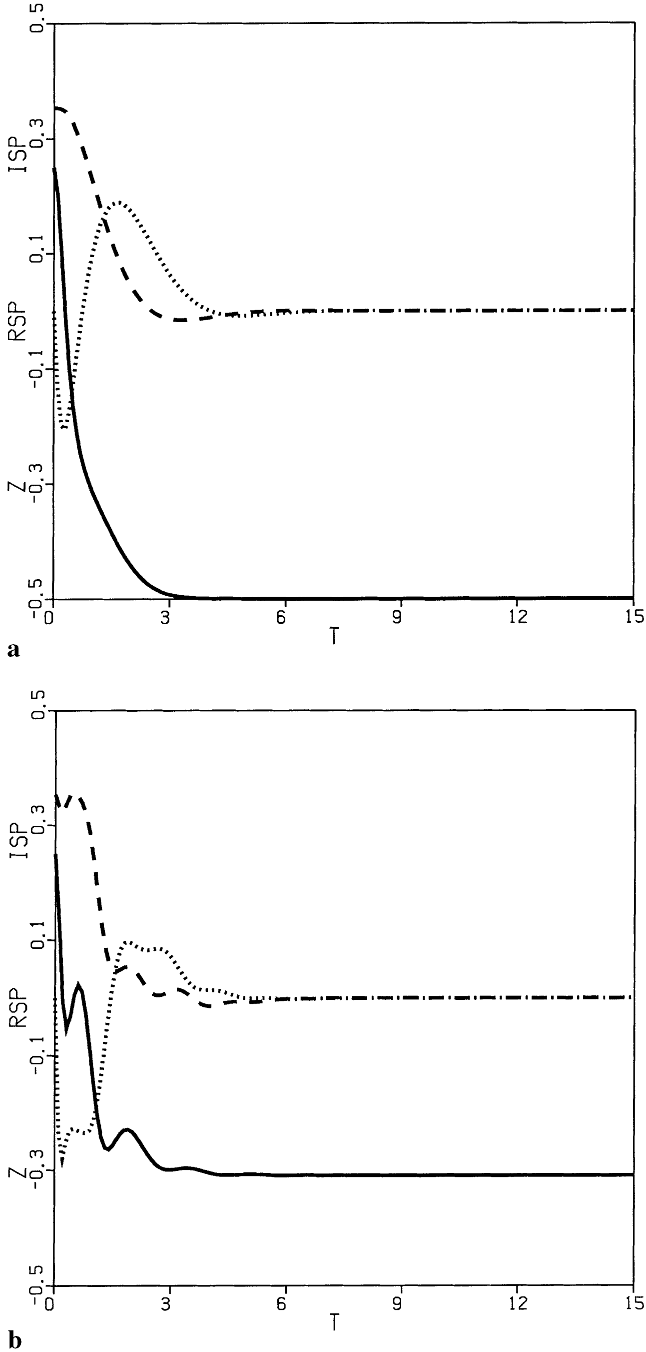

Fig. 5. Same as Fig. 2 for a damped cavity: $K = 2$

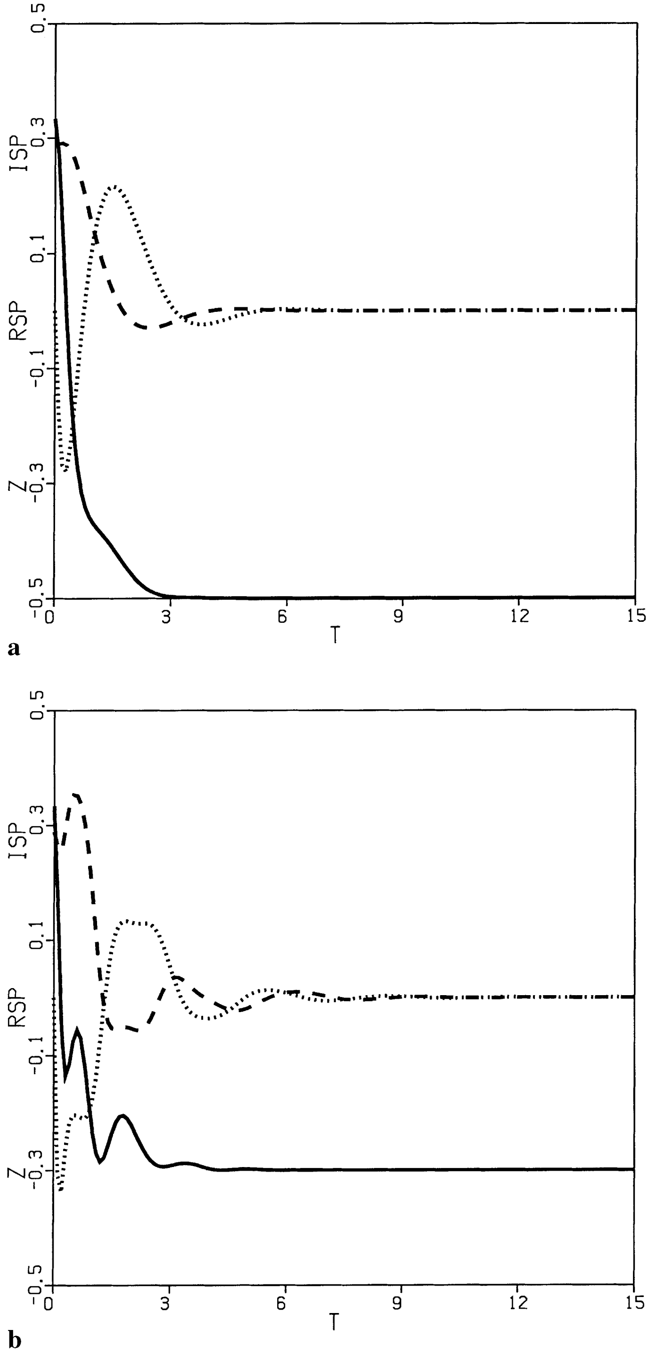

Fig. 6. Same as Fig. 3 for a damped cavity: $K = 2$

The collective radiation effects, causing a more rapid decay of the atomic population inversion, lead to the suppression of collective radiation inhibition effects, stemming from the CRT. Therefore, the new steady-state energy values, lying appreciably above their ground-state value (totally de-excited atomic system), differ only insignificantly between mono- and polyatomic cases.

In conclusion it should be noted that the experimental verification of our theoretical predictions could be perhaps possible in experiments with Rydberg atoms [1].

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